ANNEX 7

Simplified calculation of sections in the Failure Limit State under normal stresses.

1. Scope

This Annex contains simplified formulae for calculating (dimensioning or checking) box or T-sections subject to simple or straight combined bending (see Figure A.7.1). It also contains a simplified method for reducing the simple or combinedd biaxial bending of sections to straight compound bending . The expressions in this Annex are valid solely for sections made of concrete with a strength of $f_{ck} \leq 50$ N/mm^2.

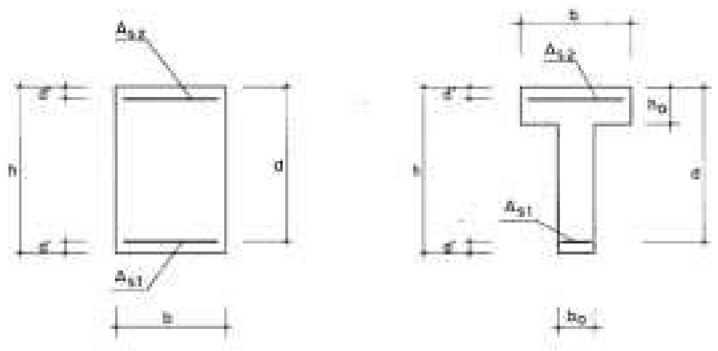


Figure A.7.1

2. Basic assumptions and limitations

The formulas presented in the following paragraphs have been determined using the basic assumptions set out in Article 42.1.2 by adopting a bilinear diagram for passive reinforcement steel and a parabolic-rectangular diagram for compressed concrete (approximated, for the calculation of stress and moment resultants, to a rectangular diagram, as set out in Article 39.5).

The failure strain domains, which identify the Failure Ultimate Limit State under normal stresses, in accordance with the criteria set out in Article 42.1.3, have also been taken into account.

The formulas indicated are valid for the various types of steel permitted in this Code for passive reinforcements, provided that these comply with:

$$\frac{d'}{d} \leq 0{,}20$$

$$\frac{d}{h} \geq 0{,}80$$

The meaning of certain variables used in the formulas in the following paragraphs is defined as follows.

$$f_{cd} = \alpha_{cc} \frac{f_{ck}}{\gamma_c}$$

$$U_0 = f_{cd}\, b\, d$$

$$U_v = 2 U_0 \frac{d'}{d}$$

$$U_a = U_0 \frac{h}{d} = f_{cd}\, b\, h$$

The equilibrium equations constitute a non-linear system due to the non-linear behaviour of materials and the existence of three pivots for defining the failure domains.

Figure A.7.2 shows, according to the position of the neutral fibre *x*, the evolution of stress in the reinforcement layers A_{s1} and A_{s2} and the evolution of the axial force and moment of the resultant of the compressed concrete about the fibres in which A_{s1} and A_{s2} are located. The definition of the moment of the resultant of the compressed block uses a reference fibre at depth *y*.

The figure and the formulas in this Annex have been determined by considering that the deformation of the yield strength of steel is ε_y = 0,002. This constitutes a reasonable simplification and an intermediate value between those corresponding to the available steels and the reduction factor of steel defined in Article 15.3.

In addition, and in order to simplify the expressions obtained, the figure of 0,0033 instead of 0,0035 has been taken as the deformation of pivot 2, i.e. the maximum deformation of compressed concrete. This assumption does not significantly affect the results obtained.

The analytical expression of the stress in the steel in layer A_{s2}, in its evolution between -f_{yd} and f_{yd}, has been linearised. This simplification leads to the definition of *-0,5 d'* and *2,5 d'* delimiters which are approximate and which also produce sufficiently accurate results.

Given these simplifications, the expressions of the various variables in Figure A.7.2 are:

- For $s_1(x) = \sigma_{s1}(x)/f_{yd}$ this gives:

$$s_1(x) = \begin{cases} -1 & -\infty < x \le x_l = 0{,}625d \\ \frac{5}{3}\frac{x-d}{x} & 0{,}625d < x \le h \\ \frac{x-d}{x-0{,}4h} & h < x \end{cases}$$

- For $s_2(x) = \sigma_{s2}(x)/f_{yd}$ this gives:

$$s_2(x) = \begin{cases} -1 & -\infty < x \le -0{,}5d' \\ \frac{2}{3}\frac{x-d'}{d'} & -0{,}5d' < x \le 2{,}5d' \\ 1 & 2{,}5d' < x \end{cases}$$

For a rectangular section, where $N_c(x)$ is the resultant of the compressed block, this gives:

$$N_c(x) = U_a\,\lambda(x)\,\eta(x)$$

and, where $M_c(x,y)$ is the benting moment of the compressed concrete block about a generic fibre situated at depth y, this gives:

$$M_c(x,y) = N_c(x)\left[y - \lambda(x)\frac{h}{2}\right]$$

where:

$$\eta(x) = 1{,}0 \qquad 0 < x < \infty$$

$$\lambda(x) = \begin{cases} 0{,}8\dfrac{x}{h} & 0 < x \le h \\ 1{,}0 - 0{,}2\dfrac{h}{x} & h < x \le \infty \end{cases}$$

The force and moment equilibrium equations, according to the above expressions, may be written as follows (see Figure A.7.3):

$$N_c(x) + U_{s1}\frac{\sigma_{s1}(x)}{f_{yd}} + U_{s2}\frac{\sigma_{s2}(x)}{f_{yd}} = N$$

$$M_c(x,d) + U_{s2}\frac{\sigma_{s2}(x)}{f_{yd}}(d - d') = N\,e_1$$

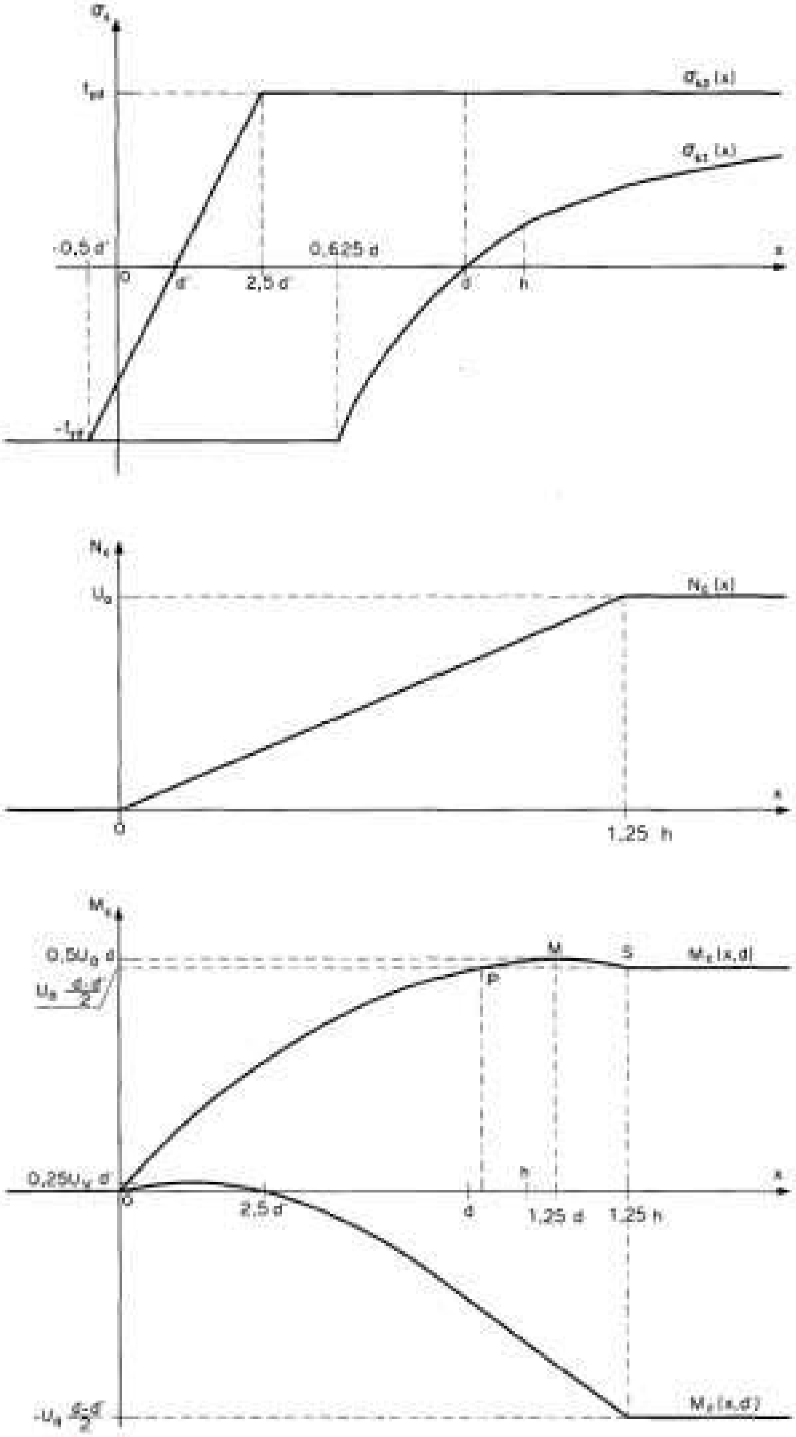
$$M_c(x,d') - U_{s1}\frac{\sigma_{s1}(x)}{f_{yd}}(d - d') = N\,e_2$$

In these expressions, the values of e_1 and e_2 are obtained as follows:

$$e_1 = e_0 - 0{,}5\,h + d$$

$$e_2 = e_0 - 0{,}5\,h + d'$$

For dimensioning, $N = N_d$ and x, U_{s1} and U_{s2} are unknown. For checking, $N = N_u$, U_{s1} and U_{s2} are data, and x and N_u are unknown.



σs
fyd
σs2 (x)
σs1 (x)
-0,5 d'
0,625 d
x
0
d'
2,5 d'
d
h
-fyd
Nc
U0
Nc (x)
0
1,25 h
Mc
0,5U0 d
M
S
Mc (x,d)
P
U0 (d-d')/2
0,25U0 d'
h
2,5 d'
d
1,25 d
1,25 h
Mc (x,d')
-U0 (d-d')/2

Figure A.7.2

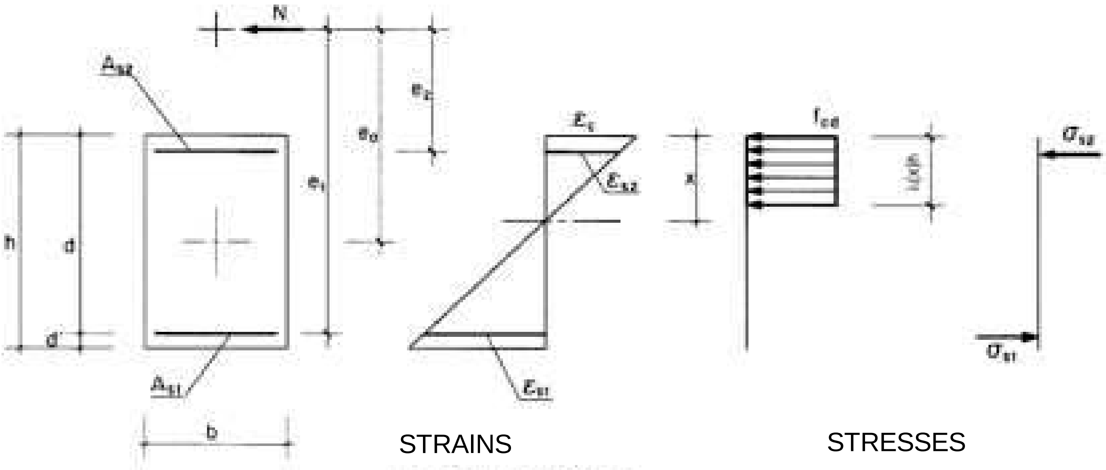


Figure A.7.3

3. Simple bending in rectangular section

3.1. Dimensioning

3.1.1. Neutral fibre confined to a prefixed depth, x_f, less than or equal to the limit depth, x_l

For concretes where $f_{ck} \leq 50$ N/mm^2, the limit depth is $x_l = 0{,}625\ d$. The frontal moment is:

$$M_f = 0.8 U_0\, x_f \left(1 - 0{,}4 \frac{x_f}{d}\right)$$

1° $M_d \leq M_f$

$$U_{s2} = 0$$

$$U_{s1} = U_0 \left(1 - \sqrt{1 - \frac{2\,M_d}{U_0\, d}}\right)$$

2° $M_d > M_f$

$$s_{2f} = \frac{2}{3}\left(\frac{x_f - d'}{d'}\right) \not> 1{,}0$$

$$U_{s2} = \frac{1}{s_{2f}}\left(\frac{M_d - M_f}{d - d'}\right)$$

$$U_{s1} = 0{,}8 U_0 \frac{x_f}{d} + \frac{M_d - M_f}{d - d'}$$

The above formulas assume that the section will only have reinforcement in the compressed face if the design bending force M_d is greater than the frontal moment, i.e. the bending of the compressed concrete block about the fibre where the tensioned reinforcement is located, for $x = x_f$.

Case 1° corresponds to dimensioning situations where $0 < x \leq x_f$. In case 2°, the position of the neutral fibre, $x = x_f$, remains constant.

The possibility of dimensioning by fixing the depth of the neutral fibre below the limit depth is useful in cases where sections must have greater ductility.

3.1.2. The prefixed fibre is located at the limit depth, x_l

1° $M_d \leq 0{,}375\ U_0 d$

$$U_{s2} = 0$$

$$U_{s1} = U_0 \left(1 - \sqrt{1 - \frac{2\,M_d}{U_0\,d}} \right)$$

2° $M_d > 0{,}375\ U_0 d$

$$U_{s2} = \frac{M_d - 0{,}375\,U_0\,d}{d - d'}$$

$$U_{s1} = 0{,}5\,U_0 + U_{s2}$$

The above formulas assume that the section will only have reinforcement in the compressed face if the design bending force M_d is greater than the limit moment $0{,}375\ U_0 d$, i.e. the bending of the compressed concrete block about the fibre where the tensioned reinforcement is located, for $x = 0{,}625\ d$, which assumes a deformation in the steel fibre of $\varepsilon_y = 0{,}002$.

Case 1° corresponds to dimensioning situations where $0 < x \leq 0{,}625\ d$. In case 2°, the position of the neutral fibre, $x = 0{,}625\ d$, remains constant.

3.2. Checking

1° $U_{s1} - U_{s2} < U_v$

$$M_u = 0{,}24\,U_v\,d' \frac{(U_v - U_{s1} + U_{s2})(1{,}5 U_{s1} + U_{s2})}{(0{,}6\,U_v + U_{s2})^2} + U_{s1}(d - d')$$

2° $U_v \leq U_{s1} - U_{s2} \leq 0{,}5 U_0$

$$M_u = (U_{s1} - U_{s2}) \left(1 - \frac{U_{s1} - U_{s2}}{2 U_0} \right) d + U_{s2}(d - d')$$

3° $0{,}5 U_0 < U_{s1} - U_{s2}$

$$M_u = \frac{4}{3} U_{s1} \left(\frac{\alpha + 1{,}2}{\alpha + \sqrt{\alpha^2 + 1{,}92 \frac{U_{s1}}{U_0}}} - 0{,}5 \right) d + U_{s2}(d - d')$$

where:

$$\alpha = \frac{U_{s1} + 0{,}6\, U_{s2}}{U_0}$$

In case 1°, the neutral fibre is positioned between $0 < x < 2{,}5d'$. In case 2°, the neutral fibre is positioned between $2{,}5\, d' \le x \le 0{,}625\, d$. In case 3°, the neutral fibre is positioned between $0{,}625\, d < x < d$.

4. Simple bending in T-section

For a T-section, the following definitions are used:

$$U_{Tc} = f_{cd}\, b\, h_0$$

$$U_{Ta} = f_{cd}\, (b-b_0)\, h_0$$

When $h_0 > 0{,}8\, d$, the depth of the neutral fibre in the rectangular block is less than h_0 and the section can be calculated as if it were a box section, $b \times h$. As a result, it is only necessary to analyse in this section the problem which arises when $h_0 < 0{,}8\, d$. This limitation must be deemed to be met in order to use the following expressions.

4.1. Dimensioning

4.1.1. Neutral fibre confined to a prefixed depth, x_f, less than or equal to the limit depth, x_l

1° $h_o \ge 0{,}8\, x_f$

The dimensioning will be carried out according to section 3.1, taking the width of the compression flange as the section width.

2° $h_o < 0{,}8\, x_f$

2°A $M_d \le U_{Tc}\, (d - 0{,}5h_0)$

As in case 1°, the dimensioning is carried out according to section 3.1, taking the width of the compression flange as the section width.

2°B $M_d \ge U_{Tc}\, (d - 0{,}5h_0)$

In this case, the dimensioning will be carried out according to section 3.1 but using an equivalent design moment, as defined below:

$$M_d^e = M_d - U_{Ta}\, (d - 0{,}5\, h_0)$$

and taking the web width as the section width and defining the mechanical capacity of the resulting reinforcement as:

$$U_{s1} = U_{s1}^e + U_{Ta}$$

$$U_{s2} = U_{s2}^e$$

where U_{s1} and U_{s2} are the mechanical capacities resulting from the dimensioning and U^e_{s1} and U^e_{s2} are the values obtained according to section 3.1 for M_d^e.

In case 1°, the depth of the compressed block will always be within the flange of the section, without involving the web.

In case 2°, dimensioning situations may be given in which the compressed block also involves the web. In case 2A, the compressed block will only be located within the flange of the section and the same expressions as for case 1° may therefore be used. In case 2B, the compressed block involves part of the web of the section but the contribution of the flanges does not vary with the position of the neutral fibre. As a result, the section can be dimensioned as if this were a box section with a width equal to that of the web, using different moment and mechanical capacity values to take account of the effect of the compression flanges.

4.1.2. The prefixed fibre is located at the limit depth, x_l

This case will be analysed according to section 4.1.1 with $x_f = x_l$.

4.2. Checking

The following non-dimensional variables are defined:

$$s_1 = s_1(1{,}25h_0) = \frac{\sigma_{s1}(1{,}25\,h_0)}{f_{yd}}$$

$$s_2 = s_2(1{,}25h_0) = \frac{\sigma_{s2}(1{,}25\,h_0)}{f_{yd}}$$

$$\beta = \frac{d}{2\,h_0} \ngtr 1{,}0$$

where:

σ_{s1} (1,25 h_0) Stress in reinforcement A_{s1} for x = 1,25 h_0

σ_{s2} (1,25 h_0) Stress in reinforcement A_{s2} for x = 1,25 h_0

1° $U_{Tc} + U_{s1}\,s_1 + U_{s2}\,s_2 \geq 0$

The section will be checked according to section 3.2, taking the width of the compression flange as the section width.

2° $U_{Tc} + U_{s1}s_1 + U_{s2}s_2 < 0$

2 A. $U_{s1} - U_{s2} \leq 0{,}5\,f_{cd}\,b_0 d + ß\,U_{Ta}$

The section will be checked according to action 3.2, taking into account the equivalent mechanical capacities of the reinforcements which are defined below:

$$U^e_{s1} = U_{s1} - U_{Ta}$$

$$U^e_{s2} = U_{s2}$$

The ultimate moment resisted by the section will be:

$$M_u = M_u^e + U_{Ta} \,(d - 0{,}5\, h_0)$$

where M_u^e is the moment obtained according to section 3.2, taking the web width as the section width and taking into account the equivalent mechanical capacities U_{s1}^e and U_{s2}^e.

2 B. $U_{s1} - U_{s2} > 0{,}5\, f_{cd}\, b_0\, d + ß\, U_{Ta}$

The section will be checked according to section 3.2, taking the web width as the section width and taking into account the equivalent mechanical capacities of the reinforcements which are defined below:

$$U_{s1}^e = U_{s1}$$
$$U_{s2}^e = U_{s2} - U_{Ta}$$

The ultimate moment resisted by the section will be:

$$M_u = M_u^e - U_{Ta} \,(0{,}5\, h_0 - d')$$

where M_u^e is the moment obtained according to section 3.2, taking the web width as the section width and taking into account the equivalent mechanical capacities U_{s1}^e and U_{s2}^e.

In case 1°, the depth of the compressed block is always contained within the flange of the section, without involving the web.

In case 2°, the web is always involved in the compressed block.

5. Dimensioning and checking of box sections subject to straight combined bending. Symmetrical reinforcement arranged in two layers with equal covers.

A simplified calculation method for box sections with two symmetrical reinforcement layers is developed below.

5.1 Dimensioning

CASE 1° $N_d < 0$

$$U_{s1} = U_{s2} = \frac{M_d}{d - d'} - \frac{N_d}{2}$$

CASE 2° $0 \leq N_d \leq 0{,}5\, U_0$

$$U_{s1} = U_{s2} = \frac{M_d}{d - d'} + \frac{N_d}{2} - \frac{N_d\, d}{d - d'}\left(1 - \frac{N_d}{2U_0} \right)$$

CASE 3° $N_d > 0,5\ U_0$

$$U_{s1} = U_{s2} = \frac{M_d}{d - d'} + \frac{N_d}{2} - \alpha \frac{U_0\, d}{d - d'}$$

With

$$\alpha = \frac{0,480\, m_1 - 0,375\, m_2}{m_1 - m_2} \ngtr 0,5 \left(1 - \left(\frac{d'}{d} \right)^2 \right)$$

where

$$m_1 = \left(N_d - 0,5U_0\right)\left(d - d'\right)$$
$$m_2 = 0,5N_d\left(d - d'\right) - M_d - 0,32U_0\left(d - 2,5d'\right)$$

5.2. Checking

CASE 1° $e_0 < 0$

$$N_u = \frac{U_{s1}(d - d')}{e_0 - 0,5(d - d')}$$

$$M_u = N_u\, e_0$$

CASE 2° $U_{S1}\,(d\text{-}d') + 0,125\ U_0\,(d + 2d'\text{-}4e_0) \leq 0$

$$N_u = \left[\sqrt{\left(\frac{e_0 - 0,5h}{d} \right)^2 + 2\frac{U_{s1}(d - d')}{U_0\, d}} - \frac{e_0 - 0,5h}{d} \right] U_0$$

$$M_u = N_u\, e_0$$

CASE 3° $U_{S1}\,(d\text{-}d') + 0,125\ U_0\ (d + 2d' - 4e_0) > 0$

$$N_u = \frac{U_{S1}(d - d') + \alpha U_0\, d}{e_0 + 0,5(d - d')}$$

$$M_u = N_u\, e_0$$

With

$$\alpha = \frac{0{,}480\, m_1 - 0{,}375\, m_2}{m_1 - m_2} \not> 0{,}5\left(1 - \left(\frac{d'}{d}\right)^2\right)$$

Where

$$m_1 = -0{,}5 U_0\, e_0 + (U_{s1} + U_{s2})\frac{d - d'}{2} + 0{,}125 U_0 (d + 2d')$$

$$m_2 = -(U_{s2} + 0{,}8 U_0)\, e_0 + U_{s2}\frac{d - d'}{2} + 0{,}08 U_0 (d + 5d')$$

6. Simple or combined biaxial bending in box section

The method proposed allows the calculation of box sections, with reinforcement at all four corners and equal reinforcements in all four faces, by reducing the problem to one of straight compound bending with a hypothetical eccentricity, as defined below (Figure A.7.4).

$$e_{y'} = e_y + \beta\, e_x \frac{h}{b}$$

where:

$$\frac{e_y}{e_x} \geq \frac{h}{b}$$

and *ß* is defined in Table A.7.6.

TABLE A.7.6

v = N_d / (bhf_{cd})	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	>0,8
ß	0,5	0,6	0,7	0,8	0,9	0,8	0,7	0,6	0,5

For high quantities (ω > 0,6), the values indicated for *ß* will be increased by 0,1 and, for small quantities (ω < 0,2), the values of *ß* will be reduced by 0,1.

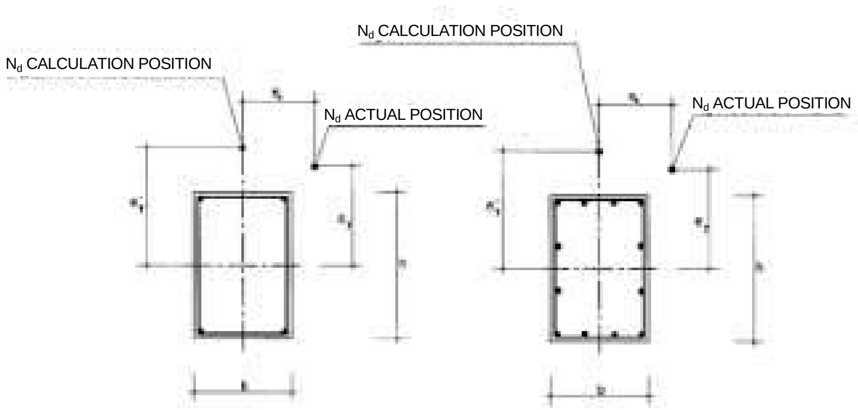


Figure A.7.6