TITLE 2. STRUCTURAL ANALYSIS

CHAPTER 5

STRUCTURAL ANALYSIS

Article 17. General

The structural analysis shall consist in determining the effects caused by the actions on all or part of the structure, for the purpose of carrying out checks on the Ultimate and Serviceability Limit States.

Article 18. Idealisation of the structure

18.1 Structural models

In order that the analysis may be carried out, both the geometry of the structure and the actions and the support conditions shall be idealised by means of a mathematical model capable of accurately reproducing the dominant structural behaviour.

The design and the arrangement of reinforcements must be consistent with the hypotheses of the calculation model with which the forces have been obtained.

18.2 Geometric data

18.2.1 Effective flange width in linear members

In the absence of a more precise determination, in T-beams it is assumed, for checks at section level, that the normal stresses are uniformly distributed over a certain reduced width of the flanges called effective width.

18.2.2 Design spans

Except where there is special justification therefore, the distance between support axes shall be considered to be the design span.

In one-way floor slabs, when the floor slab is supported on flat or mixed slabs not centred with the supports, the axis shall be taken as that passing through the centre thereof.

18.2.3 Cross-sections

18.2.3.1 General considerations

The global analysis of the structure may be carried out, in the majority of cases, using the gross sections of the elements. In some cases, when greater precision is needed in the checking of the Serviceability Limit States, net or homogenised sections may be used in the analysis.

18.2.3.2 Gross section

Gross section shall be understood to mean that resulting from the true dimensions of the member, without deducting the spaces corresponding to the reinforcements.

18.2.3.3 Net section

Net section shall be understood to mean that obtained from the gross section after deducting the longitudinal openings made in the concrete, such as piping or recesses for the passage of active reinforcements or their anchorages and the area of the reinforcements.

18.2.3.4 Homogenized section

Homogenised section shall be understood to mean that obtained from the net section specified in 18.2.3.3, taking into account the effect of the solidification of the bonded longitudinal reinforcements and the various types of concrete in existence.

18.2.3.5 Cracked cross section

Cracked section shall be understood to mean that formed by the compressed area of the concrete and the areas of the longitudinal reinforcements, both bonded active and passive, multiplied by the corresponding coefficient of equivalence.

Article 19. Calculation methods

19.1 Basic principles

Every structural analysis must satisfy the equilibrium conditions.

Unless specified to the contrary, the compatibility conditions shall always be satisfied in the Limit States considered. In cases in which the verification of compatibility is not a direct requirement, all the appropriate ductility conditions must be satisfied and an appropriate in-service behaviour of the structure guaranteed.

Generally speaking, the equilibrium conditions shall be drawn up for the original geometry of the structure without strain. For slender structures such as those defined in Article 43, the equilibrium shall be checked for the deformed configuration (second order theory).

19.2 Types of analyses

The global analysis of a structure may be carried out in accordance with the following methodologies:

- Linear analysis.
- Non-linear analysis.
- Linear analysis with limited redistribution.
- Plastic analysis.

19.2.1 Linear analysis

Is that based on the assumption of linear-elastic behaviour of the constituent materials and on the consideration of equilibrium in the non-deformed structure. In this case the concrete gross section may be used for the calculation of stresses.

The linear elastic analysis is considered, in principle, suitable for determining stresses both in Serviceability Limit States and in Ultimate Limit States in structures of all kinds, when the secondary effects are negligible, pursuant to the provisions of Article 43.

19.2.2 Non-linear analysis

Is that which takes account of the non-linear strain deformation behaviour of materials and geometric non-linearity, that is to say, satisfaction of the equilibrium of the structure in its deformed state. The non-linear analysis may be used for both Serviceability Limit State checks and Ultimate Limit State checks.

Non-linear behaviour gives rise to the invalidity of the superposition principle and, therefore, the safety format laid down in this Code is not directly applicable to non-linear analysis.

19.2.3 Linear analysis with limited redistribution

Is that in which forces are determined from those obtained by means of a linear analysis, as described in 19.2.1, and redistributions subsequently made (increases or decreases) of forces which satisfy the conditions of equilibrium between loads, forces and reactions. Redistributions of forces must be taken into account in all aspects of the design.

Linear analysis with limited redistribution may only be used for Ultimate Limit State checks.

Linear analysis with limited redistribution requires ductility conditions for the critical sections which guarantee the redistributions required for the forces adopted.

19.2.4 Plastic analysis

Is that based on a plastic, elasto-plastic or rigid-plastic behaviour of materials and which fulfils at least one of the basic theorems of plasticity: that of the lower limit, that of the upper limit or that of unicity.

It must be ensured that the ductility of the critical sections is sufficient to guarantee the formation of the collapse mechanism laid down in the design.

The plastic analysis may only be used for Ultimate Limit State checks. This method shall not be permitted when secondary effects must be taken into account.

Article 20. Structural analysis of prestressing

20.1 General considerations

20.1.1 Definition of prestressing

Prestressing shall be understood to be the controlled application of a stress to the concrete by means of the tensioning of steel tendons. The tendons shall be made of high strength steel and may consist of wires, strands or bars.

No other forms of prestressing may be considered in this Code.

20.1.2 Types of prestressing

In accordance with the position of the tendon in relation to the cross-section, the prestressing may be:

a) Internal. In this case, the tendon is positioned inside the concrete cross-section.
b) External. In this case, the tendon is positioned outside of the concrete of the cross-section and inside the depth thereof.

In accordance with the moment of the tensioning in relation to the concreting of the component, the prestressing may be:

a) With pre-tensioned reinforcements. The concreting is carried out after having provisionally tensioned and anchored the reinforcements into fixed elements. When the concrete has acquired sufficient strength, the reinforcements are freed from their provisional anchorages and, due to bonding, the force previously introduced into the reinforcements is transferred to the concrete.
b) With post-tensioned reinforcements. The concreting is carried out before the tensioning of the active reinforcements which normally are housed in ducts or sheaths. When the concrete has acquired sufficient strength, the reinforcements

shall be tensioned and anchored.

From the point of view of the bonding conditions of the tendon, the prestressing may be:

c) Bonding. This is the case of prestressing in which, in the final state, there is an adequate bonding between the active reinforcement and the concrete of the component (point 35.4.2).
d) Non-bonding. This is the case of prestressing with post-tensioned reinforcement in which reinforcement protection systems and injections which do not create bonding between this and the concrete of the component are used (point 35.4.3).

20.2 Prestressing force

20.2.1 Limitation of force

In general, the tensioning force P_0 must provide, over the active reinforcements, a stress cr$_{p0}$ not greater, at any point, than the lower of the following values:

$$0{,}70\ f_{p\ \max k}\ ; \qquad 0{,}85\ f_{pk}$$

where:

[$f_{p\ max,k}$ Characteristic maximum unitary load.
[f_{pk} Characteristic yield strength.

This stress may temporarily be increased up to the lower of the following values:

$$0{,}80\ f_{p\max k}\ ; \qquad 0{,}90\ f_{pk}$$

provided that, in anchorage the reinforcements into the concrete, a suitable reduction in stress is produced so that the limit referred to in the previous paragraph is attained.

In the case of prestressed elements with pre-tensioned reinforcement or post-tensioned elements in which both the steel for active reinforcements and the prestressing operator, or, where appropriate, the prefabricator has a quality mark, an increase of the previous values to the following shall be accepted:

a) permanent situations:

$$0{,}75\ f_{p\ \max k}\ ;\ 0{,}90\ f_{pk}$$

b) temporary situations:

$$0{,}85\ f_{p\max k}\ ;\ 0{,}95\ f_{pk}$$

20.2.2 Losses in members with post-tensioned reinforcements

20.2.2.1 Assessment of the instantaneous losses of force

Instantaneous losses of force are those which may arise during the tensioning activity and at the moment of anchorage the active reinforcements and depend on the characteristics of the structural element being studied. Their value in each section is:

$$\Delta P_i = \Delta P_1 + \Delta P_2 + \Delta P_3$$

where:

ΔP_1 Losses of force in the section being studied due to friction along the length of the pre-tensioned duct.

ΔP_2 Losses of force in the section being studied due to wedge penetration in the anchorages.

ΔP_3 Losses of force in the section being studied due to elastic shortening of the concrete.

20.2.2.1.1 Losses of force due to friction

The theoretical losses of force due to friction between the reinforcements and the sheaths or prestressed ducts depend on the total angular variation α of the route of the tendon between the section in question and the active anchorage which determines the stress in this section; of the distance x between these two sections; of the curve friction coefficient μ and the straight friction coefficient K or parasitic friction. These losses shall be assessed from the tensioning force P_0.

Losses due to friction in each section may be determined using the expression:

$$\Delta P_1 = P_0\left[1 - e^{-(\mu\alpha + Kx)}\right]$$

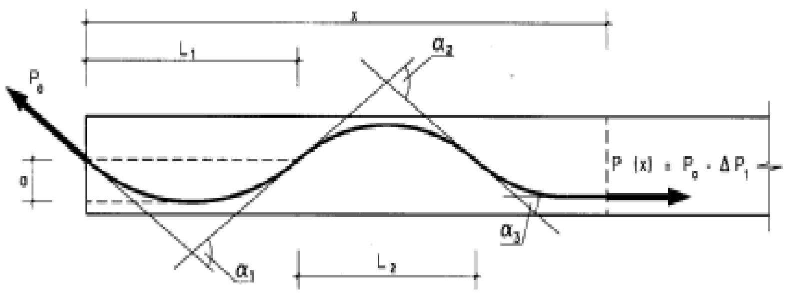
where:

μ Coefficient of friction curve.

α Sum of the absolute values of the angular variations (successive deviations), measured in radians, which describes the tendon in the distance x. It must be remembered that the routing of the tendons may be a skew curve with α having therefore to be assessed in space.

K Coefficient of parasitic friction, per linear metre.

x Distance, in metres, between the section in question and the active anchorage determining the stress therein (see Figure 20.2.2.1.1).



If $a \le 0.045\,L_1$ then α_1 may be taken as $\alpha_1 = \dfrac{8a}{L_1}$; being the error less than 5‰

$$\alpha = \sum_{i=1}^{x} \alpha_i = \alpha_1 + \alpha_2 + \alpha_3$$

α = Total angular displacement

α_i = Angular displacement in segment L_i

Figure 20.2.2.1.1

The data corresponding to the values of μ and of K must be defined experimentally, having taken account of the prestressing procedure used. In the absence of specific data, the experimental values sanctioned in practice may be used.

20.2.2.1.2 Losses due to wedge penetration

In post-tensioned straight tendons of short length, the loss of force due to wedge penetration, AP_2, may be deduced using the expression:

$$\Delta P_2 = \frac{a}{L} E_p A_p$$

where:

a Penetration of the wedge.
L Total length of the straight tendon.
E_p Longitudinal strain modulus of an active reinforcement.
A_p Section of the active reinforcement.

In other cases of straight tendons and in all cases of curved traces, the assessment of the loss of stress due to wedge penetration shall be made taking friction in the ducts into account. For this, the possible variations in μ and of K may be considered upon de-tensioning the tendon, with regard to the values appearing upon tensioning.

20.2.2.1.3 Losses due to elastic shortening of the concrete

In the case of reinforcements consisting of various tendons which have been successively tensioned, upon tensioning each tendon a new elastic shortening of the concrete is produced which unloads, in the proportional part corresponding to this shortening, to those previously anchored.

When compression stress at the level of the barycentre of the active reinforcement in the tensioning phase is significant, the value of these losses, AP_3, may be calculated, if the tendons are successively tensioned in one single operation, recognising that all tendons undergo a uniform shortening, depending on the number n thereof successively tensioned, by means of the expression:

$$\Delta P_3 = \sigma_{cp} \frac{n-1}{2n} \frac{A_p E_p}{E_{cj}}$$

where:

A_p Total cross-section of the active reinforcement.
δ_{cp} Compressive stress at the centre of gravity of the active reinforcements, produced by the force P_0 - ΔP_1 - ΔP_2 and the forces due to the actions acting at the moment of tensioning.
E_p Modulus of longitudinal elasticity of the active reinforcements.
E_{cj} Modulus of longitudinal elasticity of the concrete for the age j corresponding to the moment of applying the load to the active reinforcements.

20.2.2.2 Deferred losses of prestressing

Deferred losses are those which take place over time, after the active reinforcements have been anchored. These losses are essentially due to the shortening of the concrete by creep and shrinkage and to the relaxation of the steel of such reinforcements.

The creep of the concrete and the relaxation of the steel are influenced by the losses themselves, and, therefore, it is imperative that this interactive effect be considered.

Provided that a more detailed study of the interaction of these phenomena is not carried out, the deferred losses may be assessed in an approximated fashion in accordance with the following expression:

$$\Delta P_{dif} = \frac{n\varphi(t,t_0)\sigma_{cp} + E_p \varepsilon_{cs}(t,t_0) + 0{,}80\Delta\sigma_{pr}}{1 + n\dfrac{A_p}{A_c}\left(1 + \dfrac{A_c y_p^2}{I_c}\right)\left(1 + \chi\varphi(t,t_0)\right)} A_p$$

where:

y_p Distance from the centre of gravity of the active reinforcements to the centre of gravity of the section.

n Coefficient of equivalence = Ep/E_c.

$\varphi(t,t_0)$ Creep coefficient for a loading age equal to the age of the concrete at the time of tensioning (t_0) (see 39.8).

ε_{cs} Shrinkage strain developing after the tensioning operation (see 39.7).

σ_{cp} Stress in the concrete in the fibre corresponding to the centre of gravity of the active reinforcements due to the prestressing action, own weight and dead load.

$\Delta\sigma_{pr}$ Loss due to relaxation at constant length. This may be assessed using the following expression:

$$\Delta\sigma_{pr} = \rho_f \frac{P_{ki}}{A_p}$$

with ρ_f being the value of relaxation at constant length and infinite time (see 38.9) and A_p the total area of the active reinforcements. P_{ki} is the characteristic value of the initial prestressing force having discounted the instantaneous losses.

A_c Area of the concrete section.

I_c Inertia of the concrete section.

χ Ageing factor. In a simplified fashion, and for infinite time assessments, χ = 0.80 may be adopted.

20.2.3 Losses of force in members with pre-tensioned reinforcements

For pre-tensioned reinforcements, the losses to be considered from the moment of tensioning until the transfer of the force to the concrete are:

a) wedge penetration
b) relaxation at ambient temperature until transfer
c) additional relaxation of the reinforcement due, where appropriate, to the heating process
d) thermal expansion of the reinforcement due, where appropriate, to the heating process
e) shrinkage prior to transfer
f) instantaneous elastic shortening upon transfer.

The deferred losses subsequent to the transfer are obtained in the same way as in post-tensioned reinforcements, using the values of contraction, relaxation and creep produced after the transfer. In the assessment of deformations due to creep, the effect of the heat curing process may be taken into account through the modification of the load age of the concrete t_0 for a fictitious age t_T adjusted to temperature whose expression is:

$$t_T = \sum^{n} e^{-(4000/[273+T(\Delta t_i)]-13,65)} \Delta t_i$$

where:

t_T Age of concrete adjusted to temperature.
$T(\Delta t_i)$ Temperature in degrees °C during the period of time t_i.
$T(\Delta t_i)$ Number of days with an approximately constant temperature T.

Losses due to additional relaxation of the reinforcement due to the heating process, c), may be taken into account through the use of an equivalent time t_{eq} which should be added to the time having passed since the tensioning in the relaxation functions. In order to do this, the duration of the heating process is divided into time intervals, Δt_i, each one of these having a temperature in °C, $T_{\Delta ti}$, in such a way that the equivalent time in hours t_{eq} may be calculated as follows:

$$t_{eq} = \frac{1{,}14^{T_{max}-20}}{T_{max}-20} \sum_{i=1}^{n} (T_{\Delta t_i} - 20)\Delta t_i$$

where:

T_{max} Maximum temperature in °C reached during heat curing.

Losses due to the thermal expansion of the reinforcement due to the heating process, d), may be determined using the expression:

$$\Delta P = K \ \alpha \ E_p \ (T_{max} - T_a)$$

where:

K Experimental coefficient, to be determined in the factory and which, in the absence of tests, may be taken as $K = 0.5$.
α Thermal expansion coefficient of the active reinforcement.
E_p Modulus of longitudinal elasticity of the active reinforcement.
T_{max} Maximum temperature in °C reached during the heat curing.
T_a Mean ambient temperature in °C during manufacture.

20.3 Structural effects of prestressing

The structural effects of the prestressing may be represented using both a set of self-balanced equivalent forces and a set of imposed strains. Both methods shall give the same results.

20.3.1 Modelling of the effects of the prestressing by means of equivalent forces

The system of equivalent forces is obtained from the equilibrium of the cable and is formed by:

- Forces and moments concentrated in the anchorages.
- Forces normal to the tendons, resulting from the curvature and changes in direction thereof.
- Tangential forces due to friction.

The value of the forces and moments concentrated in the anchorages shall be deducted from the value of the prestressing force at these points, calculated in accordance with Section

20.2, from the geometry of the cable, and from the geometry of the anchorage area (see figure 20.3.1)

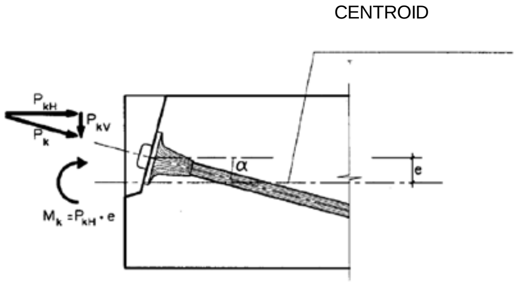


Figure 20.3.1

In the specific case of beams, with symmetry relating to a vertical plane, there shall be a horizontal component in the anchorage and another vertical component of the prestressing force and a bending moment, whose expressions shall be given by:

$$P_{k,H} = P_k \cos\alpha$$

$$P_{k,V} = P_k \, sen\alpha$$

$$M_k = P_{k,H} \, e$$

where:

α Angle formed by the route of the prestressing in relation to the directrix of the element, in the anchorage.

P_k Force in the tendon according to 20.2.

e Eccentricity of the tendon in relation to the centre of gravity of the section.

The normal forces distributed along the tendon, *n(x)*, are a result of the prestressing force and the curvature of the tendon at each point, *1/r(x).* The tangential forces, *t(x),* are proportional to the normal forces through the friction coefficient μ, according to:

$$n(x) = \frac{P_k(x)}{r(x)} \qquad ; \qquad t(x) = -\mu \, n(x)$$

20.3.2 Modelling of the effects of prestressing through imposed deformations

Alternatively, in the case of linear elements, the structural effects of prestressing may be introduced through the application of imposed deformations and curvatures that, in each section, are given by:

$$\varepsilon_p = \frac{P_k}{E_c A_c}$$

$$\left(\frac{1}{r}\right)_p = \frac{P_k \, e}{E_c I_c}$$

where:

ε_p Axial deformation due to the prestressing.

E_c Modulus of longitudinal elasticity of concrete.

A_c Area of the concrete section.

I_c Inertia of the concrete section.

e Eccentricity of the prestressing in relation to the centre of gravity of the concrete section.

20.3.3 Isostatic and statically indeterminate forces of the prestressing

The structural forces due to prestressing are traditionally defined as one of the following:

- Isostatic forces.
- Statically indeterminate forces.

The isostatic forces depend on the force of the prestressing and on the eccentricity of the prestressing in relation to the centre of gravity of the section, and may be analysed at section level. The statically indeterminate forces depend, in general, on the routing of the prestressing, on the rigidity conditions and on the support conditions for the structure and must be analysed at structure level.

The sum of the isostatic and statically indeterminate prestressing forces is equal to the total forces produced by the prestressing.

When the Limit State of Failure is observed under normal stresses of sections with bonded reinforcement, in accordance with the criteria laid down in Article 42, the design stresses must include the statically indeterminate part of the structural effect of the prestressing taking into account its value in accordance with the criteria laid down in Section 13.2. The isostatic part of the prestressing shall be considered upon assessing the strength of the section, taking into account the corresponding pre-deformation in the bonded active reinforcement.

Article 21. Flat reticular structures, one-way floor slabs and slabs

Any of the methods indicated in Article 19 may be used for the calculation of stresses in flat reticular structures.

When linear analysis with limited redistribution is used, the magnitude of the redistribution shall depend on the level of ductility in the critical sections.

Article 22. Slabs

In order for a two-way element to be considered a slab, the minimum span must be greater than four times the average width of the slab. Any of the methods indicated in Article 19 may be used for the calculation of stresses in slabs.

Article 23. Membranes and shells

Shells are superficial structural elements which, from a static point of view, are characterised by their three-dimensional resistant behaviour. Shells are usually placed under stress by the combined forces of membrane and bending, with their structural response fundamentally influenced by their geometric form, the conditions at their edge and the nature of the load applied.

Both linear and non-linear analyses are permitted for the analysis of Shells.

Plastic design is not permitted, except where duly justified in the particular case being studied.

Shells subject to compressive stresses shall be analysed taking into account possible failures due to buckling. To this end, elastic deformations shall be taken into account, and, where appropriate, those due to creep, variation in temperature and shrinkage of the concrete, the support seatings and imperfections in the form of the sheet due to inaccuracies during construction.

Article 24. D-Regions

24.1 General

D-Regions (discontinuity regions) are the structures or parts of a structure in which the general bending theory is not valid, that is to say, where the Bernoulli-Navier or Kirchhoff hypotheses do not apply. Conversely, the structures or parts of structures in which these hypotheses apply are called B regions.

D-Regions exist in a structure when there are abrupt changes in geometry (geometric discontinuity, figure 24.1.a), or in areas in which concentrated loads and reactions are applied (static discontinuity, figure 24.1.b). A D-Region may also consist of a structure in its entirety due to its form or proportions (general discontinuity). Beams of a great depth or short corbels (figure 24.1.c) are examples of general discontinuity.

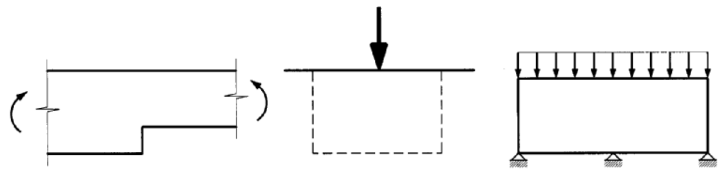


Figure 24.1.a, b and c

The following analysis methods are permitted to analyse areas of discontinuity

a) Linear analysis using elasticity theory
b) Strut-and-tie model
c) Non-linear analysis

24.1.1 Linear analysis by means of elasticity theory

The analysis shall lay down the field of main stresses and strains. The concentrations of stresses, as occur in corners or openings, may be redistributed taking into account the effects of cracking, reducing the rigidity in the corresponding areas.

Linear analysis is valid for both Service and Ultimate Limit State behaviours.

24.1.2 Strut-and-tie model

This method consists in substituting the structure, or the part of the structure forming the D-Region, with a structure with articulated bars, generally flat or in some cases spatial, representative of its behaviour. The compressed bars are called struts and represent the compression of the concrete. The tensioned bars are called ties and represent the tensile forces of the reinforcements.

The model must balance the external forces existing at the edge of the D-Region, in the case of an area within the structure, the acting external loads and the support reactions in the case of a structure with general discontinuity. This type of model, representing a perfect plastic behaviour, satisfies the requirements of the lower limit theorem of the plasticity theory, and, once the model has been decided upon, that of the unicity of the solution.

This model enables the checking of the conditions of the structure in the Ultimate Limit State for the various combinations of actions laid down in Article 13, if the conditions of the struts, ties and joints are verified in accordance with the criteria laid down in Article 40.

The checks relating to the Serviceability Limit State, in particular cracking, are not explicitly carried out, but may be considered to be satisfied if the model is directed in line with the results of a linear analysis and the conditions for the ties laid down in Article 40 are fulfilled.

24.1.3 Non-linear analysis

For a more refined analysis, the non-linear stress-strain relationships of the materials under multi-axial load states may be taken into account, using an appropriate numerical method. In this case, the analysis shall be satisfactory for both Ultimate and Serviceability Limit States.

Article 25. Analysis over time

25.1 General considerations

The analysis over time allows the structural effects of the creep, shrinkage and ageing of the concrete, and the relaxation of the prestressed steel to be obtained. These effects may be deferred deformations and displacements, as well as variations in the value or in the distribution of forces, reactions or stresses.

The analysis may be carried out by the general method laid down in Section 25.2 or the simplified methods based on the ageing factor or the like. In general, the linear viscoelasticity assumption may be applied, that is to say, proportionality between stresses and strains and overlapping over time, for compressive stresses which do not exceed 45% of the strength at the time the load is applied.

25.2 General method

For the step-by-step implementation of the general method, the following hypotheses apply:

a) The constitutive equation of concrete over time is:

$$\varepsilon_c(t) = \frac{\sigma_0}{E_c(t)} + \varphi(t,t_0)\frac{\sigma_0}{E_c(28)} + \sum_{i=1}^{n}\left(\frac{1}{E_c(t_i)} + \frac{\varphi(t,t_i)}{E_c(28)}\right)\Delta\sigma(t_i) + \varepsilon_r(t,t_s)$$

In this equation, the first term represents instantaneous deformation due to a stress applied in t_0. The second term represents the creep due to this stress. The third term represents the sum of the instantaneous deformations and creep due to the variation in stresses produced at an instant *ti.* Finally, the fourth term represents shrinkage strain.

b) For the various steels a linear behaviour under instantaneous loads shall be taken into account.
c) For prestressed steels with stresses greater than $0,5\ f_{pmax}$, relaxation and the fact that this takes place under variable deformation shall be taken into account.
d) It is considered that there is a perfect bond between the concrete and the bonding reinforcements and between the various concretes which may exist in the section.
e) In the case of linear elements, the flat deformation hypothesis for the sections is considered valid.
f) Equilibrium conditions must be verified at each section level.
g) Equilibrium must be verified at structure level taking support conditions into account.