CHAPTER 10

ULTIMATE LIMIT STATES DESIGN

Article 41. Equilibrium Limit State

It should be verified that the equilibrium limits (overturning, sliding, etc.), are not exceeded under the least favourable loading hypothesis, applying the Rational Mechanical Methods, and taking account of the actual conditions of the supports.

$$E_{d,estab} \geq E_{d,desestab}$$

in which:

$E_{d,estab}$ The effects of stabilising actions design value.
$E_{d,desestab}$ The effects of destabilising actions design value.

Article 42. Limit State at Failure under normal stresses

42.1 General design principles

42.1.1 Definition of the section

42.1.1.1 Dimensions of the section

In order to obtain strength capacity of a section, this shall be considered using its actual dimensions during the construction or service stage analysed, apart from in the case of T-beams, I-beams or similar members, when the actual widths indicated in 18.2.1 shall be taken into consideration.

42.1.1.2 Resistant section

For the purposes of the calculations for Limit State at Failure under normal stresses, the resistant section of concrete shall be obtained from the dimensions of the member and in compliance with the criteria in 40.3.5.

42.1.2 Basic hypotheses

The design of the ultimate strength capacity of sections shall be conducted on the basis of the following general hypothesis:

a) Failure is characterized by the value of strain in specified fibres of the section, defined by the failure deformation envelopes detailed in 42.1.3.
b) Strain in concrete follows a plane law. This hypothesis is valid for members in which the ratio between the distance between points of zero moment and the total depth is more than 2.
c) The strain ε_s in passive reinforcements remains equal to that of the concrete surrounding them.

The total strain in the active stress in bonded active reinforcements shall take account, not only of the strain that occurs in the corresponding fibre in the plane of strain at failure *(ε_0),* but also the strain produced by the pre-stressing and the strain of decompression (figure 42.1.2) as defined below:

$$\Delta \varepsilon_p = \varepsilon_{cp} + \varepsilon_{p0}$$

in which:

ε_{cp} Strain of decompression in the concrete at the level of the fibre in the reinforcement concerned.

ε_{p0} Pre-deformation of the active reinforcement due to pre-stressing action during the stage concerned, taking account of any losses that have occurred.

d) The stress-strain design diagram for the concrete shall be any of those defined in 39.5. The tensile strength of the concrete shall be disregarded. The design stress-strain diagram for the steel for passive reinforcements shall be as defined in 38.4. The design stress-strain diagram for steel of active reinforcements shall be as defined in 38.7.

e) The general equations of balanced forces and moments shall be applied to the stresses in the section. This is how the ultimate strength can be calculated by integrating the stresses in the concrete and in the active and passive reinforcements.

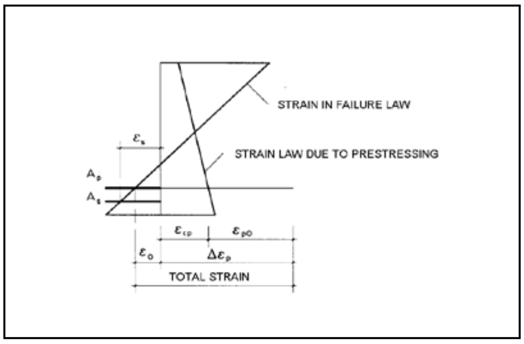


Figure 42.1.2

42.1.3 Strain envelopes

The limit stains in sections, depending on the type of stress, enable the following domains to be recognised (figure 42.1.3):

Domain 1: Pure or combined tension where the entire section is under tension. The strain lines turn about point A corresponding to an elongation in the reinforcement of the most tensioned of 10 per 1000.

Domain 2: Pure or combined bending, in which the concrete does not reach the ultimate bending strain. The strain lines turn about point A.

Domain 3: Pure or combined bending, in which the strain lines turn about point B corresponding to the ultimate bending strain of the concrete ε_{cu} defined in paragraph 39.5. The elongation of the most tensioned reinforcement is between 0.01 and ε_y with ε_y the elongation corresponding to the yield stress of the steel.

Range 4: Single or combined bending in which the strain lines turn around point B. The elongation of the most tensioned reinforcement is between ε_y and 0.

Range 4a: Combined bending in which all the reinforcements are compressed and where there is a small area of concrete in tension. The strain lines turn about point B.

Range 5: Single or combined compression in which both materials are in compression. The strain lines turn about point C, as defined by the line corresponding to the ultimate compression strain of concrete, ε_{c0}, defined in paragraph 39.5.

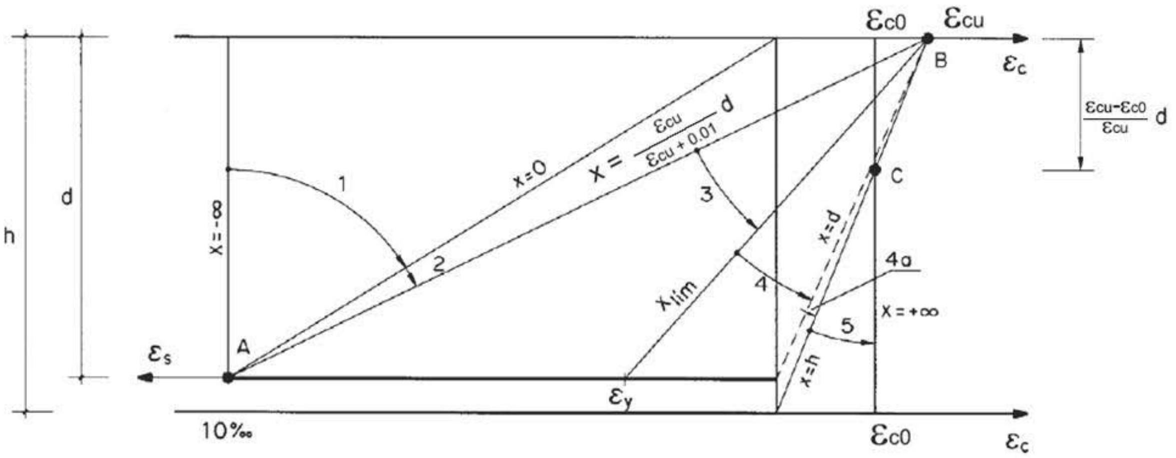


Figure 42.1.3

42.1.4 Design and verification of sections

Based on the basic hypotheses defined in 42.1.2, the equilibrium equations for the section can be established, producing a system of non-linear equations.

When design, the form and dimensions of the concrete section, the position of the reinforcement, the materials' characteristics and the design stresses are known but the plane of strain at failure and the reinforcement ratio are unknown.

When verifying, the shape and dimensions of the concrete section, the position and of the reinforcement and its ratio, and the materials' characteristics are known, and the plane of strain at failure and the sectional stresses are unknown.

42.2 Special cases

42.2.1 Minimum eccentricity

On supports and elements performing a similar function, every section submitted to an external normal compression stress N_d shall be capable of withstanding this compression, with a minimum eccentricity, due to the uncertainty at the position of the point of application of the normal stress, equal to the greater of the following values:

$h/20$ and 2 cm

This eccentricity shall be calculated starting from the centre of gravity of the gross section, and in the least favourable of the main directions and in only one direction.

42.2.2 The effect of confined concrete

Concrete confined in compression improves its strength and ductility characteristics; the latter being very important for ensuring structural performance, that permits optimum use of all the additional strength capacity of a statically indeterminate element.

Confinement of the compressed zone of concrete can be achieved with a suitable amount of transverse reinforcement that is appropriately arranged and anchored in accordance with the provisions in sub-paragraph 40.3.4.

42.2.3 Unbonded active reinforcements

The increase in tension in non-bonded active reinforcements, depends on the increase in length in the tendon between anchorages which, in their turn, depend on the overall deformation of the structure at Ultimate Limit State.

42.3 Provisions relating to reinforcements

42.3.1 General

In order for any passive reinforcements under compression to be taken into account during design, it will be necessary for them to be secured using hoops or stirrups, whose distance s_t apart and diameter ϕ_t are as follows:

s_t 15 ϕ_{min} (ϕ_{min} diameter of the thinnest compressed bar)

$\phi_t \geq$ ¼ ϕ_{max} (ϕ_{max} diameter of the thickest compressed bar)

s_t in compressed members shall always be less than the smaller dimension of the element and not exceed 30 cm.

Longitudinal passive resistant reinforcement, and skin reinforcement, shall be suitably distributed in order to avoid leaving concrete areas without any reinforcements, so that the distance between two consecutive longitudinal bars (*s*) satisfies the following limitations:

$s \leq$ 30 cm.

$s \leq$ Three times the gross thickness of the part of the member's section, flanges or webs on which they are going to be located.

In zones where the bars overlap or bend, it may be necessary to increase transverse reinforcement.

42.3.2. Pure or combined bending

Whenever failure in a section occurs due to pure or combined bending, the longitudinal tensile resistant reinforcement shall satisfy the following limitation:

$$A_p\, f_{pd} \frac{d_p}{d_s} + A_s\, f_{yd} \geq \frac{W_1}{z} f_{ct,m,fl} + \frac{P}{z}\left(\frac{W_1}{A} + e\right)$$

In which:

A_p Area of the bonded active reinforcement.
A_s Area of the passive reinforcement.
f_{pd} Design value of the tensile strength of bonded active reinforcement steel.
f_{yd} Design value of the tensile strength of passive reinforcement steel.
$f_{ct,m,fl}$ Average flexural strength of the concrete.
W_1 Section modulus of the gross section relating to the fibre under greatest tension.
d_p Depth of the active reinforcement from the most compressed fibre in the section.
d_s Depth of the passive reinforcement from the most compressed fibre in the section.
P Pre-stressing force with instantaneous losses disregarded.
A Gross concrete section area.
e Eccentricity of the pre-stressing relative to the centre of gravity of the gross section.
z Mechanical lever arm of the section. In the absence of more accurate calculations, this may be taken to be $z = 0.8\ h$.

If there is only active reinforcement in the design section, the following $\frac{d_p}{d_s} = 1$ shall be considered in the expression above.

In the case of end beam supports, apart from in one-way slabs comprising pre-cast elements, at least one third of the reinforcement necessary to withstand the maximum positive moment shall be continued as far as the supports; this shall be at least a quarter in intermediate beam supports. This reinforcement shall be extended from the centre line of the support by an amount which is the same as the net anchorage length (sub-paragraph 69.5.1).

The lower longitudinal reinforcement in slabs comprising reinforced joists shall comprise at least two bars.

42.3.3 Pure or combined compression

In sections subjected to pure or combined compression, the main reinforcements in compression, A'_{s1} and A'_{s2} (see figure 42.3.3) shall satisfy the following limitations:

$$A'_{s1}\, f_{yc,d} \geq 0.05\, N_d \qquad A'_{s1}\, f_{yc,d} \leq 0.5\, f_{cd}\, A_c$$
$$A'_{s2}\, f_{yc,d} \geq 0.05\, N_d \qquad A'_{s2}\, f_{yc,d} \leq 0.5\, f_{cd}\, A_c$$

In which:

$f_{yc,d}$ Design strength of the steel in compression $f_{yc,d} = f_{yd} > /400\ N/mm^2$.
N_d Factored normal acting compression force
f_{cd} Design value of concrete compressive strength.
A_c Area of the total concrete section.

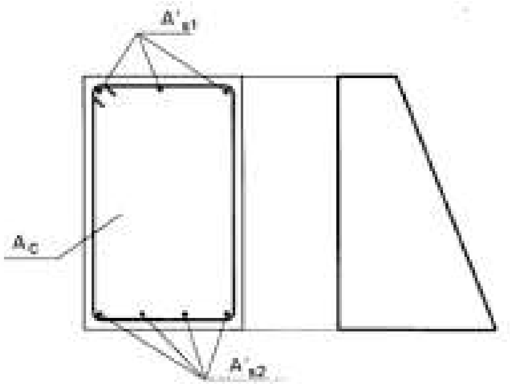


Figure 42.3.3

42.3.4 Pure or combined tension

In the case of concrete sections subjected to pure or combined tension, containing two main reinforcements, the following limitations shall be satisfied:

$$A_p\, f_{pd} + A_s\, f_{yd} \geq P + A_c\, f_{ct,m}$$

in which P is the pre-stressing force with instantaneous losses disregarded.

42.3.5 Minimum amount of reinforcement (geometric ratios)

Table 42.3.5 shows the values of the minimum geometric ratios which shall always be provided in the various types of structural elements, depending on the steel used, and provided that these values are more stringent than those indicated in 42.3.2, 42.3.3 and 42.3.4.

Table 42.3.5. Minimum geometric ratios as so many per 1,000, with reference to the total concrete section

Type of structural member		Type of steel: Steels with $f_y = 400 N/mm^2$	Type of steel: Steels with $f_y = 500 N/mm^2$
Columns		4.0	4.0
Slabs ([1])		2.0	1.8
One- way slabs	Ribs ([2])	4.0	3.0
	Distribution reinforcement perpendicular to ribs ([3])	1.4	1.1
	Distribution reinforcement parallel to ribs ([3])	0.7	0.6
Beams ([4])		3.3	2.8
Walls ([4])	Horizontal reinforcement	4.0	3.2
	Vertical reinforcement	1.2	0.9

(1) Minimum ratio of each of longitudinal and transverse reinforcements distributed along both sides. In the case of foundation slabs and footings, half of these values shall be used in each direction and arranged on the lower side.

(2) Minimum ratio with reference to a rectangular section of width b_w and a slab depth in accordance with figure 42.3.5. This ratio shall be strictly applied in ribs but not in solid areas. All joists shall have at least two longitudinal active or passive reinforcements, on their bottom flanges, symmetrical about the middle vertical plane.

(3) Minimum ratio with reference to the thickness of the in situ concrete compression layer.

(4) Minimum ratio corresponding to the side under tension. It is recommended that a minimum reinforcement of 30% of the nominal reinforcement is arranged on the opposite side.

(5) The minimum vertical ratio is the ratio corresponding to the side under tension. It is recommended that a minimum reinforcement of 30% of the nominal reinforcement is arranged on the opposite side.
Starting from a height of 2.5 m above the toe of the wall and provided that this distance is not less than half the wall height, the horizontal ratio may be reduced to 2%. If vertical contraction joints are incorporated that are 7.5 m or less apart, with the horizontal reinforcement interrupted, the minimum horizontal geometric ratios may be reduced to 2%. The minimum horizontal reinforcement shall be distributed on both sides. If both the wall's sides are visible, 50% shall be arranged on each side. In walls more than 50 cm thick, an effective area of maximum thickness 50 cm, with 25 cm distributed on each side, shall be considered, and the central area between these surface layers disregarded.

h

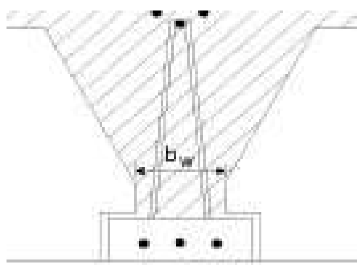


Figure 42.3.5 Rib detail.

Article 43. Instability Limit State

43.1 General

43.1.1 Definitions

For the purposes of the application of this Article 43, the following terms are used:

- *Non-sway structures* are structures whose nodes under design actions exhibit transverse displacement whose effects can be disregarded from the stability point of view of the assembly.
- *Sway structures* are structures whose nodes at design actions exhibit transverse displacement whose effects cannot be disregarded from the stability point of view of the assembly
- *Isolated supports* are statically determined supports, or supports in frames where the position of the points where the second-order moment is zero does not vary with the load size..
- *Mechanical slenderness* of a constant section support is the quotient of the effective buckling length l_0 of the support (distance between points of inflection of the column's deformed shape) divided by the radius of gyration i of the entire concrete section in the direction under consideration.
- *Geometric slenderness* of a constant section support is the quotient of the effective buckling length l_0 of the support divided by the dimension (b or h) of the section which is parallel to the plane of buckling.

Frame structures fitted with walls or wind-bracing cores and configured so that they guarantee the structure's torsional stiffness and which comply with the following, may clearly be considered as non-sway structures.

$$N_d \leq k_1 \frac{n}{n+1{,}6} \frac{\sum EI}{h^2}$$

in which:

N_d Factored vertical load which reaches the foundation with the structure fully loaded.

n Number of storeys.

h Total height of the structure from the upper foundation face.

ΣEI Sum of flexion rigidities of the counter-wind elements in the direction concerned, using the inertia of the gross section for the calculation of I.

k_i Value constant 0.62. This constant shall be reduced to 0.31 if the bracing elements cracked at Ultimate Limit State.

43.1.2 Scope

This Article relates to the verification of isolated supports and frame structures in general, in which the second order effects cannot be disregarded.

The application of this Article is limited to cases in which the effects of torsion can be disregarded.

This Code does not cover cases where the mechanical slenderness of the supports exceeds 200.

The second order effects may be disregarded in insulated supports if their mechanical slenderness is less than a limit slenderness associated with a loss of load bearing capacity in the support of 10%, with respect to a non-slender support. The lower slenderness limit λ_{inf} can be approximately calculated using the following expression:

$$\lambda_{inf} = 35 \sqrt{\frac{C}{\nu}\left[1 + \frac{0,24}{e_2/h} + 3,4\left(\frac{e_1}{e_2} - 1\right)^2\right]} \not> 100$$

In which:

ν Design value of the on-dimensional or reduced axial force actuating in the support.

$$\nu = N_d/(A_c\, f_{cd})$$

e_2 First order eccentricity in the end of the support with the larger moment, deemed to be positive.

e_1 First order eccentricity at the end of the support with the lower moment which is positive if it has the same sign as e_2.
In sway structures, e_1/e_2 shall be taken to be equal to 1.0

h Depth of the section in the bending plane considered.

C Coefficient which depends on the configuration of reinforcements whose values are:

0.24 for symmetrical reinforcement on two opposing sides in the bending plane.
0.20 for equal reinforcement on the four sides
0.16 for symmetrical reinforcement on the lateral sides.

43.2 General method

The general verification of a structure, bearing in mind geometric and mechanical non-linearities can be undertaken in accordance with the general principles indicated in 19.2. This verification justifies the fact that the structure for the various combinations of possible actions, the structure does not present any global or local instability in its constituent members, and that the strength capacity of the various sections of those elements is not exceeded.

The design shall take account of the uncertainties associated with predicting second order effects and, in particular, dimension errors and uncertainties in the position and line of action of the axial loads.

43.3 Verification of non-sway structures

The overall forces in non-sway structures may be derived according to first order theory. Based on the forces obtained in this manner, a verification shall be undertaken on the second order effects of each support considered in isolation, in accordance with 43.5.

43.4 Verification of sway structures

Sway structures shall be subject to stability verification in accordance with the general basic principles of 43.2.

For common building structures of less than 15 storeys, in which the maximum displacement at their top at characteristic horizontal loads, calculated using the first order theory and with the stiffness corresponding to gross sections, does not exceed 1/750 of the total height, each support merely needs to be verified in isolation, with the forces obtained by applying the first order theory and the buckling length in accordance with the following:

$$\alpha = \sqrt{\frac{7{,}5 + 4(\psi_A + \psi_B) + 1{,}6\,\psi_A \cdot \psi_B}{7{,}5 + (\psi_A + \psi_B)}}$$

In which:

ψ Represents the ratio of rigidities $\sum \frac{EI}{L}$ of the supports at $\sum \frac{EI}{L}$ of the beams at each end A and B of the support considered. The gross inertia of the section shall be used as the value of l.

α is the buckling length factor which shall take the following values as appropriate:

Double-fixed-end support	(l_o = 0.5 l)
Double-pinned support	(l_o = l)
Fixed-end and pinned support	(l_o =0.7 l)
Cantilever support	(l_o = 2 l)
Double-fixed-end support with moveable ends.	(l_o = l)

43.5 Verification of isolated supports

The approximate method of 43.5.1 or 43.5.2 may be used with supports with a mechanical slenderness of between λ_{inf} and 100.

The approximate method of 43.2 may be used with supports with a mechanical slenderness of between 100 and 200.

43.5.1 Approximate method. Straight combined bending

The cross-section for supports with constant section and reinforcement shall be dimensioned for a total eccentricity of:

$$e_{tot} = e_e + e_a \geq e_2$$

$$e_a = (1 + 0{,}12\beta)(\varepsilon_y + 0{,}0035)\frac{h + 20\,e_e}{h + 10\,e_e}\,\frac{l_0^2}{50\,i_c}$$

in which:

e_a Fictitious eccentricity used to represent the second order effects.

e_e Equivalent first order design eccentricity.

$e_e = 0.6e_2 + 0.4e_1 \geq 0.4e_2$ for non-sway supports;

$e_e = e_2$ for sway supports.

e_1, e_2 Eccentricities of axial force at the ends of the member defined in 43.1.2.

l_0 Buckling length.

i_c Radius of gyration of the concrete section in the direction concerned.

h Total depth of the concrete section.

ε_y Deformation in the steel for the design stress f_{yd}, i.e.,

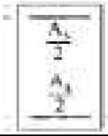
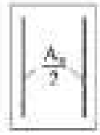
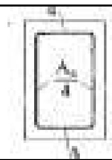
$$\varepsilon_y = \frac{f_{yd}}{E_3}$$

β Reinforcement factor obtained from

$$\beta = \frac{(d - d')^2}{4\,i_s^2}$$

with i_s being the radius of gyration of the reinforcements. The values of β and i_s are shown in table 43.5.1 for the most common reinforcement configurations.

Table 43.5.1

Reinforcement configuration	i_s^2	β
	$\frac{1}{4}(d-d')^2$	1.0
	$\frac{1}{12}(d-d')^2$	3.0
	$\frac{1}{6}(d-d')^2$	1.5
	$\frac{1}{8}(d-d')^2$	2.0

43.5.2 Approximate method. Biaxial combined bending

A separate verification may be carried out on rectangular cross-section elements constant reinforcement, along the two main planes of symmetry if the eccentricity of the axial load is located in the shaded area in figure 43.5.2.a. This situation obtains if any of the two conditions set out in figure 43.5.2.a, are satisfied, in which e_x and e_y are the design eccentricities in the direction of the x and y axes respectively.

If the conditions above are not satisfied, the support shall be deemed to be buckle-resistant, if it satisfies the following condition:

$$\frac{M_{xd}}{M_{xu}} + \frac{M_{yd}}{M_{yu}} \leq 1$$

In which:

M_{xd} is the design moment in direction x, in the critical verification section, taking account of the second order effects.

M_{yd} is the design moment in direction y, in the critical verification section, taking account of the second order effects.

M_{xu} is the maximum moment in direction x, resisted by the critical section.

M_{yu} is the maximum moment in direction y, resisted by the critical section.

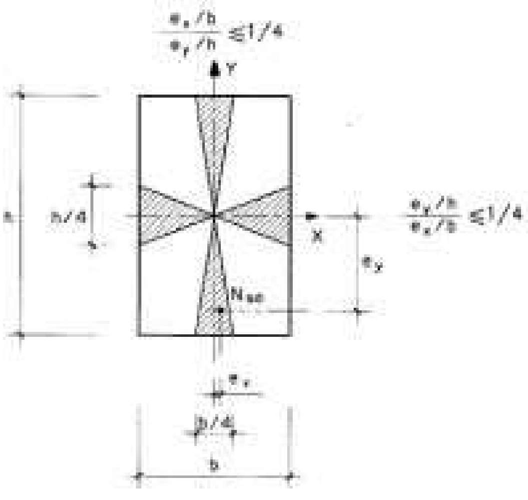


Figure 43.5.2.a

Article 44. Limit State of Failure due to shear

44.1 General considerations

When analysing the bearing capacity of concrete structures with regards to shear stresses, the general design strut-and-tie method (Articles 24 and 40) shall be used on all structural members or their parts which have planes states of stress, or similar, and which are subjected to shear actions along a known plane and which are not special cases explicitly covered by this Code, such as linear members, plates, slabs, including one-way slabs and similar structures (44.2).

44.2 Shear strength of linear members, plates, and slabs, including one-way slabs and similar structures

The requirements in the various sub paragraphs shall solely apply to linear members subjected to combined bending, shear and axial forces (in compression or tension) and to plates, slabs including basically one-way slabs.

For the purposes of this article, linear members shall be members whose distance between points of zero moment is at least twice their total depth, and whose width is no more than five times this depth, and with their main axes being straight or curved. Flat surface elements with a solid or hollow sections loaded perpendicular to their centre plane are called plates or slabs.

44.2.1 Definition of design section

For design at the Limit State of Failure due to shear stresses, sections should be considered with their actual dimensions during the phase being analysed. Except where indicated otherwise, the resistant concrete section is obtained from the actual dimensions of the member, with the criteria of 40.3.5 being met.

If the web width in the section considered is not constant, the smallest width in the section at a height equal to three quarters of the effective depth calculated from the tensioning reinforcement (figure 44.2.1.a) shall be adopted for b_0.

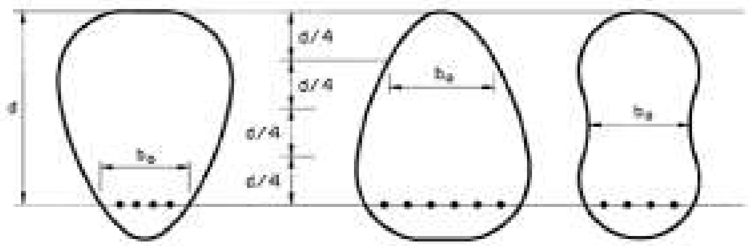


Figure 44.2.1.a

44.2.2 Effective shear force

Verifications at the Limit State of Failure due to shear may be carried out based on the effective shear stress V_{rd} obtained from the following expression:

$$V_{rd} = V_d + V_{pd} + V_{cd}$$

in which:

V_d Design value of the shear force produced by external actions.

V_{pd} Design value of the force component of the pre-stressing tendons parallel to the section under study.

V_{cd} Design value of the component parallel to the section of the resultant normal tensions, both compression and traction the passive reinforcement, on the longitudinal concrete fibres, in members with variable depth.

44.2.3 Compulsory verifications

The Limit State of Failure due to shear will be reached when either the compressive strength of the web or its tensile strength is exhausted. It is consequently necessary to verify that both the following conditions are simultaneously satisfied

$$V_{rd} \leq V_{u1}$$
$$V_{rd} \leq V_{u2}$$

in which:

V_{rt} Design value of the effective shear force in 44.2.2.
V_{u1} Ultimate shear force failure due to diagonal compression in the web..
V_{u2} Ultim

Verification of failure due to diagonal compression in the web ate shear force failure due to tension in the web $V_{rd} \leq V_{u1}$ shall be conducted at the edge of the support and not at its axis.

In members without any shear reinforcement, failure due to diagonal compression will not need to be verified.

Verification of failure due to tension in the web $V_{rd} \leq V_{u2}$ shall be carried out on a section located at a distance of one effective depth from the edge of the direct support., apart from in the case of members without any shear reinforcements in regions which do not crack in flexion, when the provisions in 44.2.3.2.1.a shall apply.

44.2.3.1 Obtaining V_{u1}

Shear stress at failure due to diagonal compression in the web shall be calculated from the following expression:

$$V_{u1} = K\ f_{1cd}\, b_0\, d\ \frac{cotg\ \theta + cotg\ \alpha}{1 + cotg^2\ \theta}$$

in which:

f_{1cd} The concrete's compression strength.

$f_{lcd} = 0.60\, f_{cd}$ for $f_{ck} \leq 60$ N/mm²
$f_{lcd} = (0.90 - f_{ck}/200)\, f_{cd} \geq 0.50\, f_{cd}$ for $f_{ck} > 60$ N/mm²

b_0 Net minimum width of the member defined in accordance with 40.3.5.

K Coefficient which depends on the axial force.

$K = 1.00$ In the case of non-pre-stressed structures or structures without any axial compression force.

$K = 1 + \frac{\sigma'_{cd}}{f_{cd}}$ for $0 < \sigma'_{cd} \leq 0.25\, f_{cd}$

$K = 1.25$ for $0.25\, f_{cd} < \sigma'_{cd} \leq 0.50\, f_{cd}$

$K = 2.5\left(1 - \frac{\sigma'_{cd}}{f_{cd}}\right)$ for $0.50\, f_{cd} < \sigma'_{cd} \leq 1.00\, f_{cd}$

In which:

σ'_{cd} Effective axial tension in the concrete (positive compression), which in columns shall be calculated bearing in mind the compression absorbed by the compressed reinforcements.

$$\sigma'_{cd} = \frac{N_d - A_{s'}\, f_{yd}}{A_c}$$

N_d Design value of the axial force (positive compression) including pre-stressing with its design value.

A_c Total concrete cross section area.

A_s' Total area of compressed reinforcement. In combined compression, it may be assumed that the entire reinforcement is subject to the tension f_{yd}.

f_{yd} Design strength of reinforcement A_s' (paragraph 40.2).

- In the case of passive reinforcements: $f_{yd} = \sigma_{sd}$
- In the case of active reinforcements: $f_{yd} = \sigma_{pd}$

α Angle between the reinforcements and the member's axis *(figure 44.2.3.1).*

θ Angle between the compression struts in the concrete and the member's axis (figure 44.2.3.1). A value which satisfies the following shall be adopted:

$$0.5 \leq cotg\ \theta \leq 2.0$$

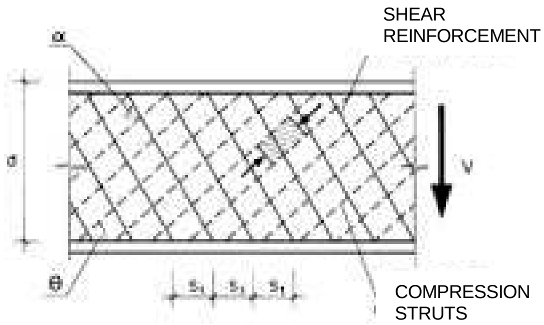


Figure 44.2.3.1

44.2.3.2 Obtaining V_{u2}

44.2.3.2.1 Members without any shear reinforcement

44.2.3.2.1.1 Members without any shear reinforcement in non-cracked regions (Md ≤ Mfis,d)

The shear strength of members with non-cracked regions and compressed webs shall be restricted according to the concrete's tensile strength, and shall be equal to:

$$V_{u2} = \frac{I \cdot b_0}{S} \sqrt{\left(f_{ct,d}\right)^2 + \alpha_l \sigma'_{cd}\ f_{ct,d}}$$

in which :

M_d Design moment of the section.

$M_{fis,d}$ Cracking moment of the section calculated using $f_{ct,d} = f_{ct,k}/\gamma_c$.

I Moment of inertia of the transverse section.

$b0$ Width of the web according to sub-paragraph 44.2.1.

S Static moment of the transverse section.

$f_{ct.d}$ Design value of the concrete's tensile strength.

σ'_{cd} Mean compression tension in the concrete due to pre-stressing force.

α_l $= l/(1.2\ l_{bd}) \leq 1$ in the case of pre-stressing tendons.
$= 1$ in the case of other types of bonded pre-stressing elements.

l_x Distance in mm of the section concerned from the start of the length of transfer .

l_{bpt} Length of transfer of the active pre-stressing reinforcement in mm, which may be adopted according to sub paragraph 70.2.3.

$l_{bpt} = \phi\ \ \sigma_p / 21$

In which:

σ_p Pre-stressing tension, after losses, in N/mm²

ϕ Diameter of the active reinforcement in mm.

This verification shall be undertaken on a section located a distance away from the edge of the support which corresponds with the intersection of the longitudinal axis passing through the centre of gravity of the section with a line at an angle of 45. starting from the edge of the support.

When determining whether the section in members comprising pre-cast elements and in situ concrete is cracked or not in flexure, (calculation of M_d and $M_{fis,d}$) its various constructional phases shall be taken into consideration; the acting loads and resistant sections shall be taken into consideration at each of these phases and the tensions corresponding to each phase superimposed.

In one-way slabs comprising pre-cast, pre-stressed joists with in situ concrete forming the other ribs and compression flange, webs are not compressed by the pre-stressing of the joists, or in any case, compression is greatly reduced and transmitted over time by creep. The ultimate shear force resisted shall therefore be the larger of the figures obtained on the basis of this article, considering the pre-stressed joists in isolation or using shear verification according to sub-paragraph 44.2.3.2.1.2.

44.2.3.2.1.2 Members without shear reinforcement in regions cracked in flexure ($M_d > M_{fis.d}$)

Ultimate shear force failure due to tensile force in the web in conventional and high strength concrete members shall be:

$$V_{u2} = \left[\frac{0{,}18}{\gamma_c}\xi(100\rho_1 f_{cv})^{1/3} + 0{,}15\sigma'_{cd}\right]b_0\, d$$

With a minimum value of:

$$V_{u2} = \left[\frac{0{,}075}{\gamma_c}\xi^{3/2} f_{cv}^{1/2} + 0{,}15\sigma'_{cd}\right]b_0\, d$$

in which :

f_{CV} Effective shear strength of the concrete in N/mm² with a value of f_{cv} = f_{ck} with f_{cv} not more than 15 N/mm² in the case of reduced concrete control, with f_{ck} being the concrete's compression strength, which, for the purposes of this paragraph, shall be considered not to exceed 60 N/mm².

$\xi = \left(1+\sqrt{\frac{200}{d}}\right) \le 2.0$ with d in mm.

d Effective depth of the cross-section with reference to the longitudinal bending reinforcement, provided that this is capable of withstanding the increase in tensile stress produced by the shear-flexure interaction (see paragraph 44.2.3.4.2).

σ'cd Mean axial tension in the web of the section (positive compression).

$$\sigma'_{cd} = \frac{N_d}{A_c} < 0{,}30\, f_{cd} \not> 12\text{MPa}$$

N_d Design value of the axial force including the pre-stressing force present in the section considered. a linear variation in the pre-stressing force may be considered in members with pre-tensioned reinforcements, from the end of the member as far as a distance equal to 1.2 times the transfer length l_{bpt}, (see 44.2.3.2.1.1). On internal supports of continuous structural with active passing reinforcement, the contribution of the pre-stressing axial force shall be disregarded when calculating N_d.

ρ_1 Geometric ratio of the main longitudinal tensioning reinforcement, whether, bonded passive or active reinforcement, anchored at least a distance, *d*, away from the section considered.

$$\rho_l = \frac{A_s + A_p}{b_0\ d} \leq 0{,}02$$

In the case of slabs with pre-cast pre-stressed joists, the ultimate shear force failure due to tensile force in the web shall be less than the values obtained by considering on the one hand the minimum width of the pre-stressed rib and, on the other hand, the smallest width of the in situ concrete above the joist, bearing in mind that the resisted shear V_{u2} shall be more than the minimum value set out in this article.

In the first case, the design value of the concrete's compressive strength shall be deemed to be that of the pre-stressed joist, the tension, σ'_{cd} shall refer to the area of the joist and the geometric ratio of the reinforcement shall refer to a reference cross-section of width b_0 and a depth *d,* with b_0 being the minimum width of the rib, and *d,* the effective depth of the slab.

In the second case, the compressive strength of the in situ concrete shall be considered to be the concrete's compressive strength, the tension σ'_{cd} shall be deemed to be zero, and the geometric ratio of the reinforcement shall refer to a cross-section of width b_0 and a depth *d,* with b_0 being the minimum width of the rib in the zone of the in situ cast concrete above the beam.

In one-way slabs comprising basic lattice reinforcement, the contribution of the lattice may be considered (in accordance with sub-paragraph 44.2.3.2.2), when verifying the shear force using the smallest width underneath the fibre corresponding to a depth of at least 20 mm below the lattice's upper round bar, as the rib width. Similarly, the rib shall be verified without the contribution of the lattice, using the rib's smaller width, between 20 mm below the lattice's upper round bar and the upper face of the slab (Figure 44.2.1.b).

44.2.3.2.2 Members with shear reinforcement

Ultimate shear force failure due to tensile force in the web shall be equivalent to:

$$V_{u2} = V_{cu} + V_{su}$$

In which:

V_{su} Contribution of the web's transverse reinforcement to shear strength.

$$V_{su} = z \text{ sen } \alpha \,(\text{cotg } \alpha + \text{cotg } \theta)\, \Sigma A_a\, f_{y\alpha d}$$

In which:

A_α Area per unit length of each set of reinforcements forming an angle α with the main axis of the member (figure 44.2.3.1)

$f_{y\alpha d}$ Design strength of the reinforcement A_α (paragraph 40.2).

- In the case of passive reinforcements: $f_{yd} = \sigma'_{sd}$
- In the case of active reinforcements: $f_{pyd} = \sigma'_{pd}$

θ Angle between the concrete's compression struts and the axis of the member (figure 44.2.3.1). The same value as that for verifying shear force failure due to diagonal compression in the web (paragraph 44.2.3.1) shall be adopted. It shall satisfy the following:

$$0.5 \leq \text{cotg } \theta \leq 2.0$$

α Angle of the reinforcements with the member's axis (figure 44.2.3.1).

z Mechanic lever arm. In pure bending and in the absence of more accurate calculations, the approximate value of $z = 0.9d$ may be adopted. In the case of in circular sections stressed in flexure, *d* may be considered to be *0.8 h.* In circular sections stressed in flexural compression, ***z*** may be approximately:

$$z = \frac{M_d + N_d z_0 - U_s^{'}(d - d')}{N_d + U_s - U_s^{'}} \begin{cases} > 0 \\ \ngtr 0.9d \end{cases},$$

in which:

z_0 Distance from the tensioned reinforcement as far as the application point of the axial force.

d, d' Distance from the most compressed fibre in the concrete as far as the centre of gravity of the tensioned and compressed reinforcement respectively.

$U_s = A_s f_{yd}$ Mechanical strength of the tensioning reinforcement.

$U'_s = A'_s f_{yd}$ Mechanical strength of the compression reinforcement.

For combined flexure and tension, 0.9d may be adopted for z.

The value of V_{su} in members reinforced with circular hoops, shall be multiplied by a factor of 0.85 to take account of the loss of efficiency of the shear reinforcement due to the transverse inclination of its constituent elements.

V_{cu} Contribution of the concrete to shear strength,

$$V_{cu} = \left[\frac{0,15}{\gamma_C} \xi (100 \rho_l f_{cv})^{1/3} + 0,15 \alpha_l \sigma'_{cd} \right] \beta b_0 d$$

In which:

f_{cv} Effective shear strength of the concrete in N/mm^2 with a value of $f_{cv} = f_{ck}$ with f_{cv} not exceeding 15 N/mm^2 in the case of reduced concrete inspection.

f_{ck} Compression strength of the concrete in N/mm^2. Values of f_{ck} of up to 100 N/mm^2 shall be adopted.

and in which:

$$\beta = \frac{2 \cot g\ \theta - 1}{2 \cot g\ \theta_e - 1} \qquad si\ 0,5 \leq \cot g \theta < \cot g\ \theta_e$$

$$\beta = \frac{\cot g\ \theta - 2}{\cot g\ \theta_e - 2} \qquad si\ \cot g\ \theta_e \leq \cot g\ \theta \leq 2,0$$

θ_e Reference angle of inclination of cracks, for which either of the following two values may be adopted:

a) Simplified method. θ_e is the angle corresponding to the inclination of the cracks in the web of the member at the time of cracking, calculated from the following expression:

$$\cot g\ \theta_e = \frac{\sqrt{f_{ct,m}^2 - f_{ct,m}(\sigma_{xd} + \sigma_{yd}) + \sigma_{xd}\sigma_{yd}}}{f_{ct,m} - \sigma_{yd}} \begin{cases} \geq 0,5 \\ \leq 2,0 \end{cases}$$

$f_{ct,m}$ Mean tensile strength of the concrete (paragraph 39.1).

σ_{xd} σ_{yd} Design values of normal tensions at the centre of gravity of the section parallel to the main axis of the member and at shear stress V_d respectively. The tensions σ_{xd} and σ_{yd} shall be obtained from the design actions, including the pre-stressing tensions in accordance with the theory of elasticity and assuming non-cracked concrete and considering the tensile stresses to be positive.

b) General method. The angle θ_e in sixtieths of a degree can be obtained by considering the interaction with other Ultimate Limit States forces, whose value in degrees may be obtained from the following expression:

$$\theta_e = 29 + 7\varepsilon_x$$

in which:

ε_x Longitudinal strain in the web (figure 44.2.3.2.2) expressed as so many per thousand, and obtained using the following equation:

$$\varepsilon_x \approx \frac{\frac{M_d}{z} + V_{rd} - 0{,}5N_d - A_p\sigma_{p0}}{2\left(E_s A_s + E_p A_p\right)} \cdot 1000 \not< 0$$

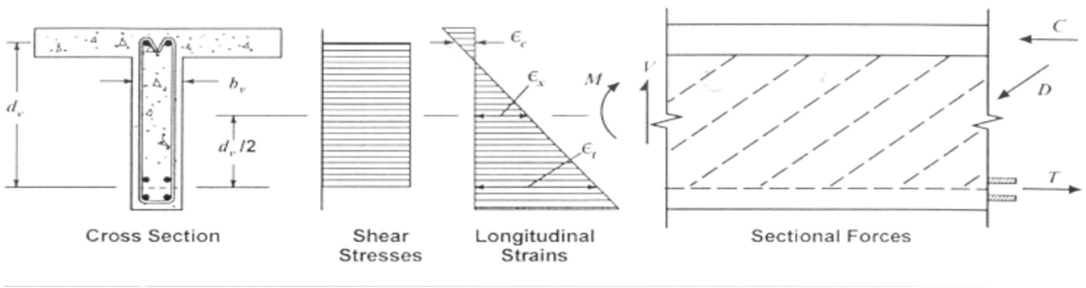


Figure 44.2.3.2.2

σ_{P0} Tension in the pre-stressing tendons, when the surrounding concrete's strain is 0.

The following considerations shall be taken into account when calculating the value of the longitudinal strain in the web, ε_x,:

a) V_{rd} *and* M_d shall be taken as positive, and M_d shall be taken to be not less than z-V_{rd}.
b) N_d shall be deemed to be positive compression.
c) The values of A_s and A_p are those of the reinforcement anchored in the section considered. If this is not the case, it shall be reduced in proportion to its anchorage length shortfall.
d) If tensile stress can produce cracking in the compressed flange, the value of ε_x obtained in the equation shall be doubled.

44.2.3.3 Special load cases

When a beam is subjected to a hanging load applied at a level whereby it lies outside the compression flange of the beam, suitable transverse reinforcement and suspension reinforcement shall be arranged and suitable anchored in order to transfer the corresponding load to the compression flange.

Additionally, the end zones of pre-stressed members, especially in case of pre-tensioned active reinforcements anchored by bonding, it will be necessary to examine the progressive transfer of the pre-stressing force to the member, by assessing this force in each section..

44.2.3.4 Arrangements for reinforcements

44.2.3.4.1 Transverse reinforcements

The longitudinal distance s_t between transverse reinforcements (figure 44.2.3.1) shall satisfy the following conditions, in order to ensure suitable confinement of the concrete subjected to diagonal compression:

$$s_t \leq 0.75\, d\, (1+\text{cotg}\alpha) \leq 600 \text{ mm} \qquad \text{if } V_{rd} \leq \frac{1}{5} V_{ul}$$

$$s_t \leq 0.60\, d\, (1+\text{cotg}\alpha) \leq 450 \text{ mm} \qquad \text{if } \frac{1}{5} V_{ul} < V_{rd} \leq \frac{2}{3} V_{ul}$$

$$s_t \leq 0.30\, d\, (1+\text{cotg}\alpha) \leq 300 \text{ mm} \qquad \text{if } V_{rd} > \frac{2}{3} V_{ul}$$

In bent bars this spacing shall never exceed the value of 0.6 d (1+cotα). The transverse spacing $s_{t,trans}$ between transverse reinforcements shall satisfy the following condition:

$$s_{t,trans} \leq d \leq 500 \text{ mm}$$

If there is compression reinforcement present and this is taken into consideration in the calculation, hoops or stirrups shall also satisfy the requirements in Article 42.

In general, the linear elements shall include cross-sectional reinforcement in an effective manner.

Hoops and stirrups shall also be extended for a length that is equal to half the depth of the member beyond the section in which they theoretically cease to be necessary. Hoops and stirrups in supports shall be arranged close to their edges.

Shear reinforcements shall form an angle with the axis of the beam of between 45. and 90., sloping in the same direction as the main tensile stress produced by external loads at the centre of gravity of the section of the beam which is assumed to be not cracked.

Bars forming transverse reinforcement may be active or passive and both types can be arranged in an isolated or combined manner. Their minimum reinforcement ratio shall satisfy the following:

$$\sum \frac{A_\alpha\, f_{y\alpha,d}}{sen\ \alpha} \geq \frac{f_{ct,m}}{7{,}5}\, b_0$$

At least one third of the shear reinforcement necessary and in every case, the minimum ratio indicated shall be installed in the form of stirrups forming an angle of 90. with the centre line of the beam. However, in ribbed one-way slabs with a depth not exceeding 40 cm, basic lattice reinforcements may be used for the shear reinforcement whenever a prefabricated footing is used or if the rib is completely concreted in situ.

44.2.3.4.2 Longitudinal reinforcements

The longitudinal reinforcements for bending shall be capable of withstanding an increase in tension on top of that produced by M_d, of:

$$\Delta T = V_{rd}\, cotg\ \theta - \frac{V_{su}}{2} (\, cotg\ \theta + cotg\ \alpha\ )$$

This specification is automatically met if the curve of design moments M_d is offset by an amount equal to:

$$s_d = z \left(cotg\ \theta - \frac{1}{2} \frac{V_{su}}{V_{rd}} (\, cotg\theta + cotg\ \alpha\ ) \right)$$

in the least favourable direction. (figure 44.2.3.4.2).

If no shear reinforcement is present, V_{su} shall be taken to be 0 in the expressions above.

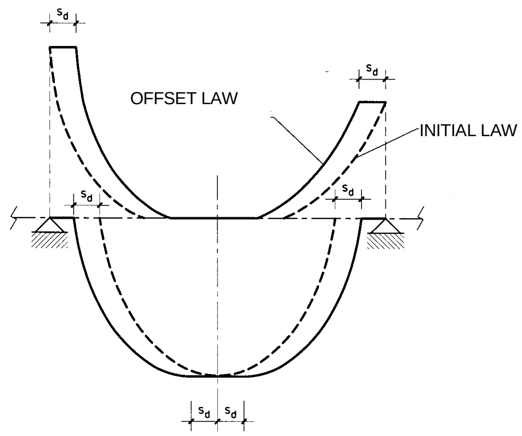


Figure 44.2.3.4.2

44.2.3.5 Longitudinal shear between the flanges and web of a beam

The strut and ties method shall be used to calculate the reinforcement connecting the flanges and the web of T-beams, I-beams and box girders and similar, (Article 40).

When determining the longitudinal shear force plastic redistribution may be assumed in a beam area of length a_r (figure 44.2.3.5.a).

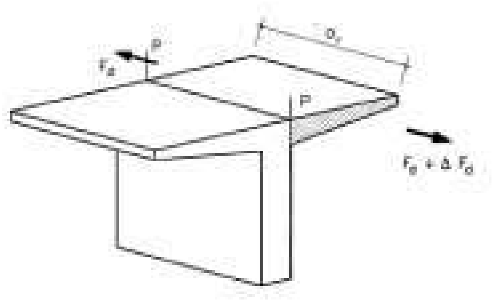


Figure 44.2.3.5.a

The mean longitudinal shear force per unit length which must be resisted shall be:

$$S_d = \frac{\Delta F_d}{a_r}$$

in which:

a_r Length of plastic redistribution concerned. The law of moments in the length, a_r shall exhibit a monotonously increasing or decreasing variation. At least the points of moment sign change shall always be adopted as zone a_r limits.

ΔF_d Variation in the distance a_r of the longitudinal shear force acting on the section of the flange outside the plane P.

In the absence of more rigorous calculations, the following shall be satisfied:

$$S_d \leq S_{u1}$$

$$S_d \leq S_{u2}$$

In which:

S_{u1} Ultimate longitudinal shear force due to diagonal compression in the plane P.

$$S_{u1} = 0.5\, f_{1cd}\, \boldsymbol{h_0}$$

In which:

f_{1cd} The concrete's compression strength (sub-paragraph 40.3.2) with a value of:

- In the case of compressed flanges:
  - $f_{1cd} = 0.60\, f_{cd}$ for $f_{ck} \leq 60\ N/mm^2$
  - $f_{1cd} = (0.90 f_{ck}/200)\, f_{cd}$ for $f_{ck} > 60\ N/mm^2$

- In the case of tensioned flanges:
  - $f_{1cd} = 0.40\, f_{cd}$ in the case of tensioned flanges.

$\boldsymbol{h_0}$ Thickness of the flange in accordance with 40.3.5.

S_{u2} Ultimate longitudinal shear force due to tension in plane P.

$$S_{u2} = S_{su}$$

In which:

S_{su} Contribution of the reinforcement perpendicular to the plane P to the shear strength.

$$S_{su} = A_p\, f_{yp,d}$$

A_p Reinforcement per unit length perpendicular to plane P (figures 44.2.3.5. b and c).

$f_{yp,d}$ Design strength of the reinforcement A_p:

$f_{yp,d} = \sigma_{sd}$ in the case of passive reinforcements

$f_{yp,d} = \sigma_{pd}$ in the case of active reinforcements

Where there is longitudinal shear between flanges and webs, combined with transverse bending, the reinforcements necessary under both headings shall be calculated and the total of these two used; the longitudinal shear reinforcement may be reduced, bearing in mind compression due to transverse bending.

4.4.2.3.6 Vertical shear at joints in hollow core slabs

The vertical shear force per unit length in longitudinal joints in hollow core slabs comprising in situ concrete, Vd, (Figure 44.2.3.6) shall not exceed the resisted shear force Vu, calculated as being the smaller of the following values:

$$V_u = 0{,}25\left(f_{bt,d} \cdot \sum h_f + f_{ct,d} \cdot h_t\right)$$
$$V_u = 0{,}15\, f_{ct,d}\left(h + h_t\right)$$

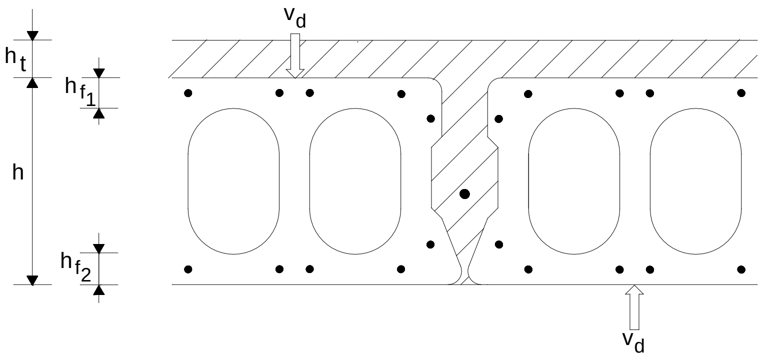


Figure 44.2.3.6 Shear force in joints of pre-stressed hollow core slabs

In which:

$f_{bt,d}$ Design tensile strength of the concrete in the precast slab.

$f_{ct,d}$ Design tensile strength of in situ concrete.

Σh_f Sum of the smaller thicknesses of the upper and lower flanges of the precast slab (figure 44.2.3.6).

h Net height of the joint.

h_t Thickness of the concrete of in situ cast upper slab.

44.2.3.7 Punching in one way slabs

The slab's puncture resistance shall be verified if large concentrated loads are to be applied,.

Slabs subjected to large concentrated loads shall be provided with an upper in situ cast slab and subjected to a special study.

The point load on the precast hollow core slab of in pre-cast hollow core slabs without any in situ cast upper slab shall not exceed:

$$V_d=b_w h(f_{ctd}+0,3\text{-}a\text{-}a_{cpm})$$

In which:

b_w Effective width, obtained from the sum of the flanges affected in accordance with figure 44.2.3.7.

h Total height of the slab.

$f_{ct,d}$ Design tensile strength of the concrete in the precast slab.

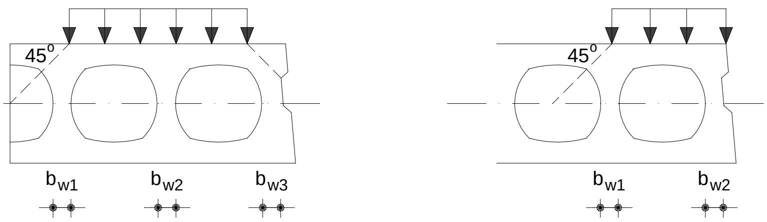
σ_{cPm} Mean tensile stress in the concrete due to the pre-stressing force.

α Coefficient equal *to* $[xl(1.2\text{-}/_{bpt})] \leq 1.$

In which:

x Distance from the section to the end.

l_{bpt} Transfer length of the active pre-stressing reinforcement (paragraph 70.2.3).



a) General situation

b) Free slab edge

Figure 44.2.3.7. Effective width in pre-stressed hollow core slabs

In the case of concentrated loads of which more than 50% are acting on a free edge of the slab with a width of b_w (see figure 44.2.3.7.b), the strength value obtained from the formula is only applicable if a minimum of one wire or strand is installed in the external flange plus a transverse passive reinforcement. If either of these two conditions is not satisfied, the strength must be divided by a factor of 2.

Plates or bars shall be fitted in the upper part of the element as the transverse passive reinforcement; they shall be at least 1.20 m long, be fully anchored and designed to resist a tensile force that is equal to the total concentrated load.

If there is a load with a width of less than half the width of the cavity on any cavity, a second strength value shall be calculated using the previous formula, but replacing *h* with the smaller thickness of the upper flange, and b_w with the width of the loaded zone. When verifying, the smaller of the strength values calculated above shall be adopted.

Article 45 Limit State of Failure due to torsion in linear elements

45.1 General considerations

The requirements in this article shall solely apply to linear elements subjected to pure torsion or the combined stresses of torsion and both shear and axial bending.

For the purposes of this article, linear elements shall be deemed to be elements whose distance between points of zero moment is at least two and a half times their total depth, and whose width is no more than four times that depth with their main axes being straight or curved.

The two-dimensional bending states (m_x, m_y and m_{xy}) in slabs and plates shall be dimensioned in accordance with Article 42, taking account of the main directions of the forces and the directions in which the reinforcement is arranged.

When the static equilibrium of a structure depends on the torsional resistance of one or more of its elements, these shall be dimensioned and verified in accordance with this article. When the static equilibrium of the structure does not depend on the torsional resistance strength of one or more of its elements, this limit state will merely need to be verified in elements whose torsional stiffness has been considered in the forces design.

45.2 Pure torsion

45.2.1 Definition of the design section

The torsional resistance of sections shall be calculated using a thin walled closed section. This means that solid sections shall be replaced by equivalent thin walled sections. Sections of complex shapes, such as T-beams, shall be divided into several sub-sections, each of which shall be modelled as an equivalent thin walled section, and their total torsional resistance shall be calculated as the sum of the resistances of the various members. Sections shall be divided in order to maximise calculated stiffness. In areas near to supports, their elements whose transmission of stresses to the support elements cannot take place directly shall be not considered as collaborating to the torsional stiffness of the section.

The effective thickness, h_e of the wall of the design section (figure 45.2.1) shall be:

$$h_e \leq \frac{A}{u}\begin{cases} \leq h_o \\ \geq 2c \end{cases}$$

in which:

A Area of the transverse section inscribed in the external circumference including inner void areas.

u External circumference of the transverse section.

h_o Actual thickness of the wall in the case of hollow sections.

c Covering of longitudinal reinforcements.

A value of h_e less than *A/u,* may be used provided that it satisfies the minimum conditions indicated, and so that the concrete compression requirements set out in 45.2.2.1 can be satisfied.

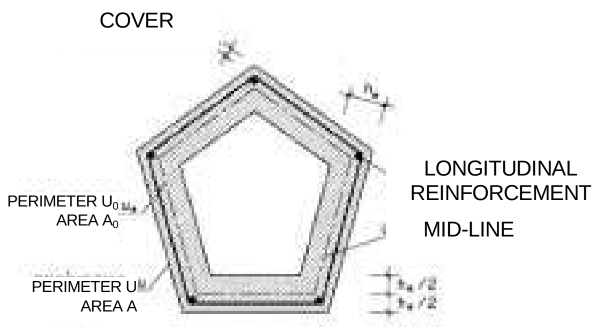


Figure 45.2.1

45.2.2 Verifications that should be performed

The Limit State of Failure due to torsion will be reached when either the compressive strength of the concrete or the tensile strength of the reinforcement arrangement is exhausted.

It is consequently necessary to verify that both the following conditions are simultaneously satisfied:

$$T_d \leq T_{u1}$$
$$T_d \leq T_{u2}$$
$$T_d \leq T_{u3}$$

In which:

T_d Design value of the torsional moment for the section.
T_{u1} Maximum torsional moment which the concrete's compressed struts can resist.
T_{u2} Maximum torsional moment which transverse reinforcements can resist.
T_{u3} Maximum torsional moment which longitudinal reinforcements can resist.

Torsional reinforcements are assumed to comprise a transverse reinforcement formed from continuous hoops, located in planes perpendicular to the member's main axis. Longitudinal reinforcements shall comprise passive or active reinforcement parallel to the member's main axis, distributed uniformly and not more than 30 cm apart on the external circumference of the effective hollow section or in a double layer in the external circumference and on the inside of the effective or actual hollow section. At least one longitudinal bar shall be located on each corner of the actual section, to ensure the transmission of the longitudinal forces exerted by the compression struts to the transverse reinforcement.

45.2.2.1 Obtaining T_{u1}

The ultimate torsional force that compressed struts can resist may be obtained from the following expression:

$$T_{u1} = 2K\alpha\, f_{1cd}\, A_e\, h_e \frac{\text{cotg}\,\theta}{1+\text{cotg}^2\,\theta}$$

in which:

f_{1cd} Concrete's compression strength.

$f_{1cd} = 0.60\, f_{cd}$ for $f_{ck} \leq 60$ N/mm²
$f_{1cd} = (0.90 - f_{ck}/200)\, f_{cd} \geq 0.50\, f_{cd}$ for $f_{ck} > 60$ N/mm²

K Coefficient which depends on the axial force defined in sub-paragraph 44.2.3.1.
α 0.60 if there are stirrups only along the external circumference of the member;
0.75 if closed stirrups are installed on both faces of the wall of the equivalent hollow section or the actual hollow section.

θ Angle between the concrete's compression struts and the member's axis. The expressions in Article 44 may be used to obtain this angle. A value that is consistent with the value adopted for verifying the Ultimate Limit State of failure due to shear and which satisfies the following shall be adopted:

$$0.50 \leq cotg\ \theta \leq 2.00$$

A_e Area enclosed by the middle line of the design effective hollow section (figure 45.2.1).

45.2.2.2 Obtaining T_{u2}

The torsional stress which transverse reinforcements can resist is obtained from:

$$T_{u2} = \frac{2\ A_e\ A_t}{s_t}\ f_{yt,d}\ cotg\ \theta$$

in which:

A_t Area of the reinforcements used as hoops or transverse reinforcement.

s_t Longitudinal spacing between hoops or bars in the transverse reinforcement.

$f_{yt,d}$ Design strength of the reinforcement steel A_t (paragraph 40.2).

- For passive reinforcements: $f_{yt,d} = \sigma_{sd}$
- For active reinforcements: $f_{yt,d} = \sigma_{pd}$

45.2.2.3 Obtaining T_{u3}

The tensional stress which longitudinal reinforcements can resist may be calculated using:

$$T_{u3} = \frac{2\ A_e}{u_e}\ A_l\ f_{yl,d}\ tg\ \theta$$

in which:

A_1 Area of the longitudinal reinforcements.

f_{y1d} Design strength of the steel in the longitudinal reinforcements A_1 (paragraph 40.2).

- For passive reinforcements: $f_{y1,d} = \sigma_{sd}$
- For active reinforcements: $f_{y1,d} = \sigma_{pd}$

u_e Circumference of the middle line in the design effective hollow section A_e (figure 45.2.1).

45.2.2.4 Warping caused by torsion

In general, the stresses produced by the co-action of torsional warping may be disregarded in the design of linear concrete members.

45.2.3 Provisions relating to reinforcements

The longitudinal distance between torsional stirrups s_t shall not exceed:

$$s_t \leq \frac{u_e}{8}$$

and shall satisfy the following conditions to ensure suitable confinement of the concrete subjected to diagonal compression:

$$s_t \leq 0{,}75\ a\ (1+\text{cotg}\alpha) \leq a \not> 600\ \text{mm}\quad si\ \ T_d \leq \frac{1}{5}\ T_{u1}$$

$$s_t \leq 0{,}60\ a\ (1+\text{cotg}\alpha) \leq a \not> 450\ \text{mm}\quad si\ \ \frac{1}{5}\ T_{u1} < T_d \leq \frac{2}{3}\ T_{u1}$$

$$s_t \leq 0{,}30\ a\ (1+\text{cotg}\alpha) \leq a \not> 300\ \text{mm} \quad si\ T_d > \frac{2}{3}\ T_{u1}$$

with *a* being the smaller dimension of the sides, making up circumference u_e.

45.3 Interaction between torsion and other stresses

45.3.1 General method

The same procedure as for pure torsion (45.2.1) shall be used, in order to define a design effective hollow section. The perpendicular and tangential stresses produced by the forces acting on this section shall be calculated using the conventional elastic or plastic methods.

Once the tensions have been determined, the reinforcements necessary in any wall in the design effective hollow section may be determined using the plane tension distribution formulae. The main compressive stress in the concrete can also be determined. If the reinforcements obtained from this method are not feasible or appropriate, the tensions obtained in any zone may be replaced by a system of equivalent static forces, and these may be used in the reinforcement. In this case, the consequences that this change has in unusual areas such as hollows or beam ends shall be verified.

The main compressive stresses σ_{cd} calculated in the concrete, in the various walls of the design effective hollow section, shall satisfy:

$$\sigma_{cd} \leq 2\alpha\ f_{1cd}$$

in which α and f_{1cd} are defined in 45.2.2.1. and 40.3, respectively.

45.3.2 Simplified methods

45.3.2.1 Torsion combined with bending and axial loads.

The longitudinal reinforcements necessary for torsion and flexural compression or flexural tension shall be calculated separately, assuming that both types of load are acting independently. The reinforcements determined in this way shall be combined in accordance with the following rules:

a) In the area stressed due to combined bending, the longitudinal reinforcements for torsion shall be added to those required for bending and axial stresses.

b) In the area compressed due to combined bending, if the tensile stress generated solely by the torsional force is greater than the compressive force acting on that area due to combined bending, a longitudinal reinforcement capable of resisting this difference shall be incorporated. If this is not the case, it shall be verified whether it is necessary to incorporate a compressed longitudinal reinforcement, whose ratio may be determined using the following expression:

$$\rho_l \cdot f_{yd} = \sigma_{md} - \alpha \cdot f_{cd} \cdot \left[0.5 + \sqrt{0.25 - \left(\frac{\tau}{\alpha \cdot f_{cd}} \right)^2} \right] \geq 0$$

In which:

p_L Ratio of longitudinal reinforcement per unit length to be added in the compression zone of the effective hollow section due to the effect of the torsional moment.

$$\rho_L = \frac{\Delta A_s^{'}}{s\ h_e}$$

σ_{md} Mean compression tension in the concrete present in the compressed zone of the effective hollow section due to the design bending and axial forces (M_d, N_d) acting concomitantly with the design torsional stress (T_d)

τ Tangential torsional stress:

$$\tau = \frac{T}{2 \cdot A_e \cdot h_e}$$

A value not exceeding 400 N/mm^2 shall be adopted for the steel's design strength.

In every case, it shall be verified that $T_d \leq T_{u1}$ in accordance with sub-paragraph 45.2.2.1

45.3.2.2 Torsion combined with shear

The concomitant design torsional and shear stresses shall satisfy the following condition in order to ensure that the concrete is not subject to excessive compression:

$$\left(\frac{T_d}{T_{u1}} \right)^{\beta} + \left(\frac{V_{rd}}{V_{u1}} \right)^{\beta} \leq 1$$

in which :

$$\beta = 2\left(1 - \frac{h_e}{b} \right)$$

b Width of the element, which is the same as the total width of a solid section and the sum of the widths of webs of box type sections.

The calculations for the design of stirrups shall be undertaken separately; for torsion: in accordance with 45.2.2.2; for shear: in accordance with 44.2.3.2.2. In both calculations, the same angle θ shall be used for the compression struts. The reinforcements calculated in this way shall be added together, taking account of the fact that the torsion reinforcements shall be arranged on the outer circumference of the section, which is not compulsory for shear reinforcements.

If the section in one-way slabs comprising pre-stressed hollow core elements is subject to concomitant shear and torsional loads, the shear strength V_{u2n} shall be calculated on the basis of:

$$V_{u2n} = V_{u2} - V_{Td}$$

With

$$V_{Td} = \frac{T_d}{2b_w} \cdot \frac{\Sigma b_w}{b - b_w}$$

in which:

V_{u2n} Net value of shear strength

V_{u2} Shear strength according to sub-paragraph 44.2.3.2.

V_{Td} Increase in shear produced by the torsional moment.

T_d Design torsional moment in the section studied

b_w Width of the external web at the centre of gravity (see Figure 45.3.2.2)

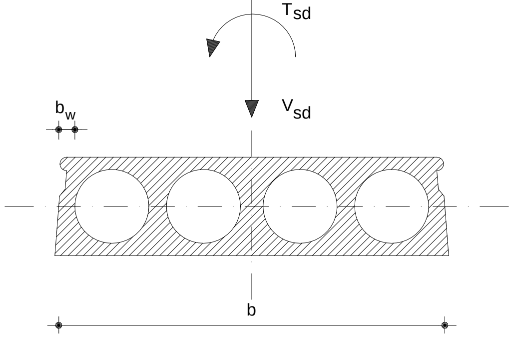


Figure 45.3.2.2. Shear and torsional load or eccentric shear

Article 46. Punching Limit State

46.1 General considerations

Resistance to transverse forces produced by concentrated loads or reactions acting on slabs without any transverse reinforcement shall be verified using a nominal tangential tension on a critical surface concentric to the loaded zone.

46.2 Critical punching shear surface

The critical surface or area is defined as a distance *2d* away from the perimeter of the loaded area or the support, with *d* being the effective depth of the slab calculated as half the sum of the effective depths corresponding to the reinforcements in two orthogonal directions.

The critical area is calculated as the product of the critical perimeter u_1 and the effective depth, *d.* The critical perimeter u_1 is determined in accordance with figures 46.2.a, 46.2.b and 46.2.c in the case of internal supports and from an edge or corner respectively.

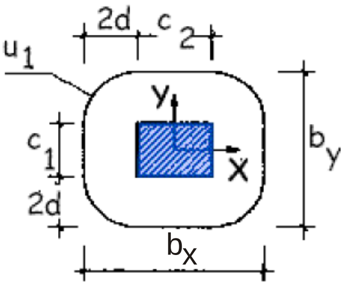


Figure 46.2.a

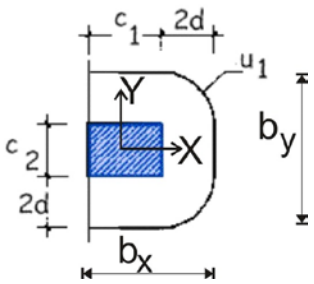


Figure 46.2.b

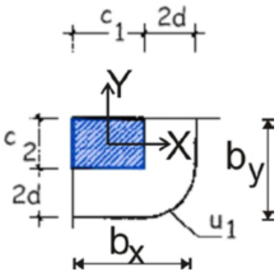


Figure 46.2.c

In other supports or loaded areas, the critical perimeter is determined on the basis of their enclosing line according to figure 46.2.d. If there are openings, hollows or lightweight elements in the slab (such as pots or blocks) located less than *6d* away, the area between the tangents on the voids marked from the centre of gravity of the column or loaded area, shall be deducted from u_1f in accordance with figure 46.2.e.

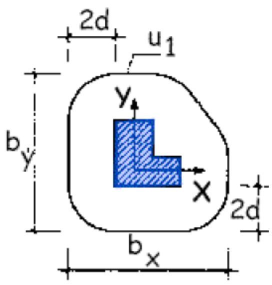


Figure 46.2.d

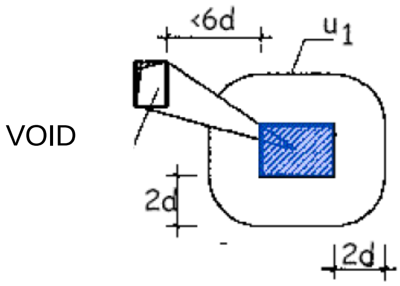


Figure 46.2.e

46.3 Slabs without any reinforcement for punching shear

No reinforcement for punching shear will be necessary if the following condition is satisfied:

$$\tau_{sd} \leq \tau_{rd}$$

In which:

τ_{sd} Design nominal tangential stress in the critical perimeter.

$$\tau_{sd} = \frac{F_{sd,ef}}{u_1 \, d}$$

$F_{sd.ef}$ Design effective punching shear stress, taking account of the effect of the moment transferred between the slab and the support.

$$F_{sd,ef} = \beta\, F_{sd}$$

β Coefficient that takes account of the effects of eccentricity of the load. If there are no moments transferred between the slab and the support, the value of 1.00 shall be adopted. In other words, where there are moments transferred between the slab and the supports, β may be adopted as being 1.15 in internal supports, 1.40 on edge supports, and 1.50 on corner supports.

F_{sd} Design punching shear . This shall be obtained as the support reaction, with external loads and the equivalent pre-stressing forces in the direction opposite to this reaction, which act on the perimeter located *h/2* away from the section of the support or loaded area, being able to be deducted.

u_1 Critical perimeter defined in figures 46.2.a, 46.2.b, 46.2.c, 46.2.d, 46.2.e.

d Effective depth of the slab.

τ_{rd} Maximum tension resistance in the critical perimeter:

$$\tau_{rd} = \frac{0{,}18}{\gamma_c}\,\xi\,\left(100\,\rho_\ell\, f_{cv}\right)^{1/3} + 0{,}1 \cdot \sigma'_{cd}$$

with a minimum value of:

$$\tau_{rd} = \frac{0{,}075}{\gamma_c}\,\xi^{3/2} f_{cv}^{\;1/2} + 0{,}1 \cdot \sigma'_{cd}$$

f_{cv} Effective shear strength of the concrete in N/mm^2 with value $f_{cv}= f_{ck}$ with f_{cv} no more than 15 N/mm^2 in the case of reduced concrete inspection, with f_{ck} being the concrete's compressive strength, which, for the purposes of this paragraph, shall be considered not to exceed 60 N/mm^2.

p_ℓ Geometric ratio of the slab's main longitudinal tensioning reinforcement, including any bonded active reinforcement, and calculated using:

$$\rho_\ell = \sqrt{\rho_x\,\rho_y} \le 0{,}02$$

with p_x and p_y being the ratios in two perpendicular directions. The ratio to be considered in each direction is that existing in a width equal to the dimension of the support plus *3d* on either side of the support, or as far as the edge of the slab, in the case of an edge or corner support.

$\xi = 1 + \sqrt{\dfrac{200}{d}} \le 2{,}0$ with *d* in mm

σ'_{cd} Mean axial tension in the critical surface being verified (positive compression). This shall be calculated as the mean of the tensions in the two directions σ'_{cdx} and σ'_{cdy}.

$$\sigma'_{cd} = \frac{\left(\sigma'_{cdx} + \sigma'_{cdy}\right)}{2} < 0{,}30 \cdot f_{cd} \not> 12\ N/mm^2$$

$$\sigma'_{cdx} = \frac{N_{d,x}}{A_x}\ ;\ \sigma'_{cdy} = \frac{N_{d,y}}{A_y}$$

When σ'_{cd} is obtained from pre-stressing, this shall be calculated bearing in mind the pre-stressing force that actually reaches the critical perimeter, and considering the co-actions applied to the deformation in the slab by vertical elements.

$N_{d,x}$,$N_{d,y}$Longitudinal forces at the critical surface emanating from a load or pre-stressing.
A_x, A_y Surface areas defined by sides b_x and b_y in accordance with sub-paragraph 46.2.

$$A_x = b_x \cdot h \quad \text{and} \quad A_y = b_y\ h$$

46.4 Slabs with reinforcement for punching shear

When reinforcement for punching shear is necessary, three verifications shall be carried out: one in the area with transverse reinforcements in accordance with 46.4.1, one in the zone outside the punching shear reinforcement according to 46.4.2, and one in the zone adjacent to the support or load in accordance with 46.4.3.

46.4.1 Zone comprising transverse punching shear reinforcement

Vertical stirrups or bars bent to an angle of α,shall be incorporated in the area comprising punching shear reinforcements and calculated so that the following equation is satisfied:

$$\tau_{sd} \leq 0{,}75\tau_{rd} + 1{,}5 \cdot \frac{A_{sw}\, f_{y\alpha,d} \operatorname{sen}\alpha}{\mathrm{s} \cdot \mathrm{u}_1}$$

in which:

r_{sd} Design nominal tangential stress according to 46.3.
r_{rd} Tensile strength in the critical perimeter obtained using the expression of 46.3, but using the actual value of f_{ck}.
A_{sw} Total area of punching shear reinforcement in a perimeter concentric with the support or loading area, in mm^2.
s Distance in a radial direction between two concentric reinforcement perimeters. (figure 46.5.a), in mm or between the perimeter and the face of the support, if there is only one.
$f_{y\alpha,d}$ `Design strength of the reinforcement A_a in N/mm^2, not exceeding 400 N/mm^2.

46.4.2 Zone outside punching shear reinforcement

Verification will need to be carried out that no reinforcement is needed in the zone outside the punching shear reinforcement.

$$F_{sd,ef} \leq \left(\frac{0{,}18}{\gamma_c} \xi \left(100\, \rho_\ell\, f_{ck} \right)^{1/3} + 0{,}1\sigma'_{cd} \right) u_{n,ef} \cdot d$$

in which:
$u_{n,ef}$ Perimeter defined in figure 46.5.1.
p_ι Geometric ratio of longitudinal reinforcement intersecting perimeter $u_{n,ef}$ calculated, as indicated in 46.3.
f_{ck} The compressive strength of the concrete in N/mm^2. For calculation purposes no value exceeding 60 N/mm^2 shall be adopted.
σ'_{cd} Mean axial stress in the perimeter u_n,ef, calculated in the same manner as in 46.3, and adopting for $N_{d,x}$, $N_{d,and}$ the value of the longitudinal forces in that perimeter, due to a load or pre-stressing.
A_x, A_y Surface areas defined by sides b_x and b_y in accordance with figure 46.5.a:

$$A_x = b_x \cdot h \quad \text{and} \quad A_y = b_y\ h$$

At the distance where this condition can be verified, it shall be assumed that the effect of the moment transferred between the support and the slab by tangential stresses has disappeared, so that $F_{sd,ef}$ shall be calculated, using $\beta = 1$, in accordance with sub-paragraph 46.3.

46.4.3 Zone adjacent to support or load

It shall be verified that the maximum punching shear satisfies the following limitation:

$$\frac{F_{sd,ef}}{u_0\, d} \leq 0{,}5\, f_{1cd}$$

in which:

f_{1cd} Compressive strength of the concrete.

$f_{1cd} = 0.60\, f_{cd}$ for $f_{ck} \leq 60$ N/mm²

$f_{1cd} = (0.90 - f_{ck}/200)\ \ f_{ck} \geq 0.50\, f_{ck}$ for $f_{ck} > 60$ N/mm²

u_0 Verifying perimeter (figure 46.4.3):

- In internal supports, u_0 is the perimeter of the support's transverse section.
- In edge supports:

$$u_0 = c_1 + 3d \leq c_1 + 2_{c2}$$

in which c_1 and c_2 are the dimensions of the support.

- In corner supports:

$$u_0 = 3d \leq c_1 + c_2$$

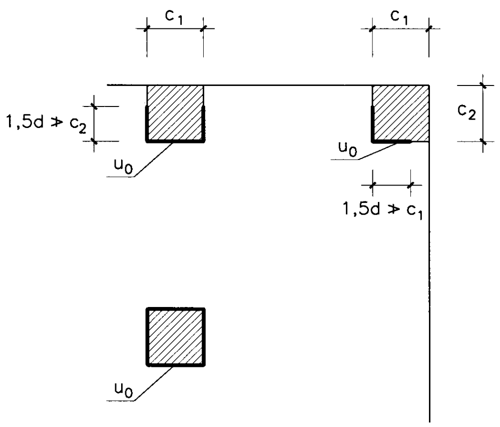


Figure 46.4.3. Critical perimeter u_0

When calculating $F_{sd,ef}$ based on F_{sd}, the values of β set out in 46.3 shall be adopted.

46.5 Provisions relating to reinforcements

The punching shear reinforcement shall be defined in accordance with the following criteria:

- The punching shear reinforcement shall comprise hoops, vertical shear assemblies or bent bars.
- The constructive configurations in plan shall satisfy the specifications in figure 46.5.a.
- The constructive configurations in elevation shall satisfy the specifications in figure 46.5.b.
- The punching shear reinforcement shall be anchored from the centre of gravity of the compressed block and underneath the longitudinal tensioning reinforcement. The anchorage of the punching shear reinforcement shall be carefully examined especially in thin slabs.

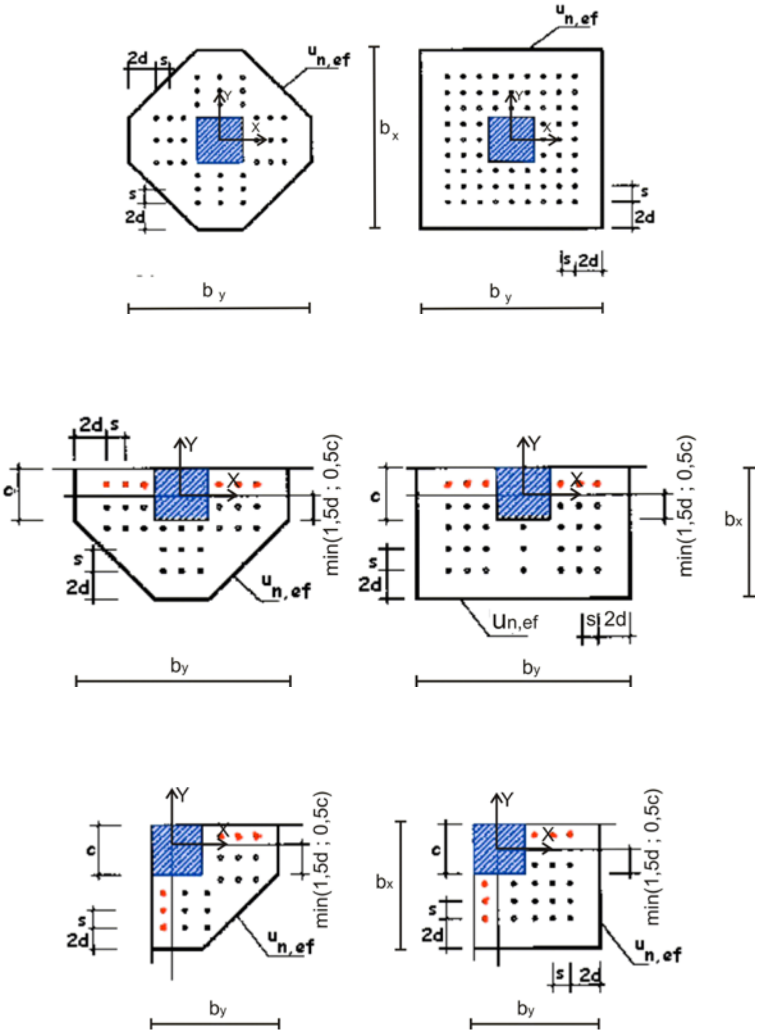


Figure 46.5 a. Plan view of types of punching shear reinforcement. The darker areas indicate the reinforcement necessary. The lighter areas indicate additional reinforcement.

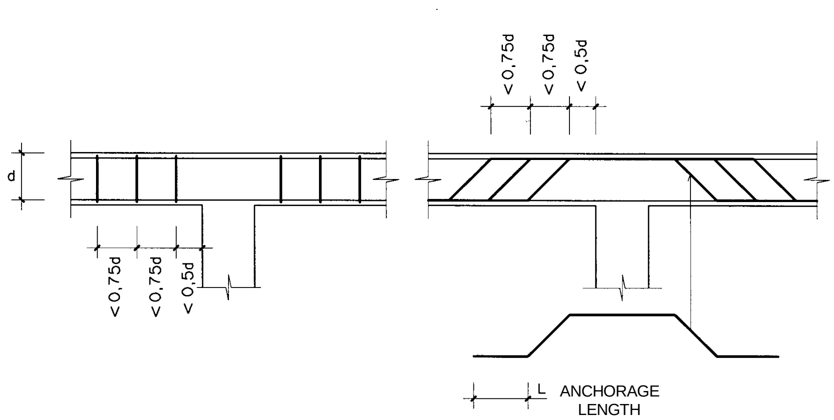


Figure 46.5. b. Elevation of the types of punching shear reinforcement

Article 47. Limit State of Failure due to longitudinal shear forces at joints between concretes

47.1 General

The Limit State dealt with in this article is that caused by the longitudinal shear stress produced by the tangential action to which a joint between concretes is subjected.

The design longitudinal shear stress, τr,d shall be calculated on the basis of the variation in the sum of the blocks of perpendicular stresses along the element in tension ΔT or compression ΔC. This variation along the member shall be calculated in sections corresponding to one effective depth at the height of the contact surface. In order to obtain the design longitudinal shear stress, the variation in the sum of the blocks (ΔC or ΔT) shall be uniformly distributed on the contact surface corresponding to the perimeter p and a length which is the same as the effective depth of the member d:

$$\tau_{r,d} = \frac{\Delta C \text{ ó } \Delta T}{p\, d}$$

The greater value of *d* and *0.8-h* shall be adopted as the design length in pre-stressed members.

47.2 Longitudinal shear stress resistance at joints between concretes

Verifying the ultimate limit state due to longitudinal shear stress shall be verified by:

$$\tau_{r,d} \leq \tau_{r,u}$$

with:

$\tau_{r,u}$ Failure due to longitudinal shear stress corresponding to the ultimate limit state for longitudinal shear strength, according to the information indicated below and assuming that the minimum mean thickness of the concrete on either side of the joint is 50 mm measured perpendicularly to the plane of the joint and with a minimum thickness of 30 mm being permitted locally.

47.2.1 Sections without any transverse reinforcement

The ultimate longitudinal shear stress, $\tau_{r,u}$ shall have the following value:

$$\tau_{r,u} = \beta\left(1{,}30 - 0{,}30\,\frac{f_{ck}}{25}\right) f_{ctd} \not< 0{,}70\beta f_{ctd}$$

in which:

β Factor assigned the following values:

0.80 in rough contact surfaces of composite sections that are interconnected so that one composite section may not overhang the other, for example, are dovetailed, and if the surface is open and rough, e.g. like joists as left by a floor laying machine.

0.40 in intentionally rough surfaces with a high degree of roughness.

0.20 in unintentionally rough surfaces with a low degree of roughness.

f_ck Characteristic compressive strength of the weakest concrete in the joint.
$f_{ct,d}$ Design tensile strength of the weakest concrete in the joint.

The values for the contribution from cohesion between concrete members, β *(l.30-0.30* f_{ck}*/25)* $f_{ct,d}$ at low fatigue or dynamic stresses shall be reduced by 50%.

Where tensions perpendicular to the contact surface obtain (for example, loads hanging from the bottom face of a composite beam) the contribution from the cohesion between concrete members shall be considered to be zero. ($\beta f_{ct,d} = 0$).

47.2.2 Sections with transverse reinforcement

47.2.2.1 Sections with $\tau_{r,d} \leq 2{,}5\beta\left(1{,}30 - 0{,}30\frac{f_{ck}}{25}\right)f_{ctd}$

The ultimate longitudinal shear stress $\tau_{r,u}$ shall have the following value:

$$\tau_{r,u} = \beta\left(1{,}30 - 0{,}30\frac{f_{ck}}{25}\right)f_{ct,d} + \left(\frac{A_{st}}{sp}f_{y\alpha,d}(\mu\, sen\alpha + \cos\alpha) + \mu\,\sigma_{cd}\right) \leq 0{,}25\, f_{cd}$$

in which:

f_{ck} Characteristic compressive strength of the weakest concrete in the joint.
$f_{ct,d}$ Design tensile strength of the weakest concrete in the joint.
A_{st} Cross-section of effectively anchored steel bars, closing the joint.
s Distance between the closing bars along the joint plane.
p Contact surface per unit length. This shall not extend to zones where the penetrating width is less than 20 mm or the maximum diameter of the edge or with a cover of less than 30 mm.
$f_{y\alpha,d}$ Design strength of transverse reinforcements in N/mm^2 ($\not>$ 400N/mm^2).
α Angle formed by the joining bars with the plane of the joint. Reinforcements with α > 135° or α < 45° shall not be incorporated.
σ_{cd} External design tensile stress perpendicular to the plane of the joint.
σ_{cd} > 0 for compression tensions. (If σ_{cd} < 0, β $f_{ct,d}$ = 0).

The values for the contribution from cohesion between concrete members, β *(l.30-0.30* f_{ck}*/25)* $f_{ct,d}$ at low fatigue or dynamic stresses shall be reduced by 50%.
Where tensile stresses perpendicular to the contact surface obtain (for example suspended loads on the lower base of a composite beam, the contribution from cohesion between concrete members shall be considered to be zero. ($\beta f_{ct,d} = 0$).

47.2.2.2 Sections with $\tau_{r,d} > 2{,}5\beta\left(1{,}30 - 0{,}30\frac{f_{ck}}{25}\right)f_{ctd}$

The ultimate longitudinal shear stress τ_{ru} shall have the following value:

$$\tau_{r,u} = \left(\frac{A_{st}}{sp}f_{y\alpha,d}(\mu\, sen\alpha + \cos\alpha) + \mu\,\sigma_{cd}\right) \leq 0{,}25\, f_{cd}$$

Table 47.2.2.2

Values of β and μ coefficients as a function of the type of surface

		Type of surface	
		Low degree of roughness	High degree of roughness
β		0.2	0.8
μ	$\tau_{r,d} \le 2{,}5\beta\left(1{,}30 - 0{,}30\dfrac{f_{ck}}{25}\right)f_{ctd}$	0.3	0.6
	$\tau_{r,d} > 2{,}5\beta\left(1{,}30 - 0{,}30\dfrac{f_{ck}}{25}\right)f_{ctd}$	0.6	0.9

The contribution of the joining reinforcement to the joint's longitudinal shear strength in the section considered shall only be calculated if the geometric ratio of the transverse reinforcement satisfies the following:

$$\frac{A_{st}}{sp} \ge 0{,}001$$

47.3 Provisions for reinforcements

A brittle joint is defined as a joint whose geometric connecting reinforcement ratio is less than the value indicated in paragraph 47.2 in order for the contribution of the joint reinforcement to be taken into account, and a ductile joint is one whose connecting reinforcement ratio exceeds this value.

In brittle joints, the distribution of the connecting reinforcement shall be rendered proportional to the law of shear stresses. In ductile joints the tension redistribution along the joint hypothesis may be assumed, although it is also advisable to distribute the connecting reinforcement proportionally to the law of shear stresses.

In the case of members stressed at significant dynamic loads, connecting reinforcements shall always be placed in cantilevers and the end quarters of spans.

The gaps between transverse reinforcements connecting contact surfaces shall not exceed the smaller of the following values:

- Depth of the composite section.
- Four times the smaller dimension of members forming the joint.
- 60 cm.

Connecting reinforcements in the contact zones shall be suitably anchored on both sides of the joint.

Article 48. Fatigue Limit State

48.1 Principles

It may be necessary in structural elements subjected to variable significant repeated actions to verify that the effect of these actions does not compromise their safety during the anticipated service period.

The safety of an element or a structural detail with regard to fatigue is guaranteed if it satisfies the general conditions set out in 8.1.2. The concrete and steel shall be verified separately.

It is not usually necessary to verify this Limit State in normal structures.

48.2 Compulsory verifications

48.2.1 Concrete

For fatigue purposes, the maximum compression stress values produced either by perpendicular or tangential stresses (compressed struts), due to dead and live loads caused by fatigue shall be limited.

In the case of elements subjected to shear without any transverse reinforcement, their strength due to the effect of fatigue shall also be limited.

The maximum compression tension and shear strength values shall be defined in accordance with existing experiments or, as appropriate, with the opposing criteria indicated in the technical literature.

48.2.2 Active and passive reinforcements

In the absence of more stricter criteria, based for example on the theory of mechanical fracture, the maximum tensile stress variation, $\Delta\sigma_{sf}$, due to the imposed loads caused by fatigue (13.2), shall not be less than the fatigue limit, $\Delta\sigma_{d}$, defined in 38.10.